

Chapter 4

Gamma ray attenuation coefficient

4.1 Linear attenuation Coefficient

The probability for a photon to pass through a medium without any kind of interaction is the product of the probabilities of survival for each particular type of interaction. Hence if I_0 is the initial intensity of a narrow beam of gamma ray, its intensity I after passing through an attenuator with thickness x is given by equation (1.1)

$$I = I_0 e^{-\sigma x} e^{-\tau x} e^{-kx} \quad (4.1)$$

where $e^{-\sigma x}$, $e^{-\tau x}$ and e^{-kx} are the probabilities of survival to pass through a distance x in the absorber without any coherent or incoherent collision, photoelectric absorption and pair production collision respectively. Here,

$$\sigma + \tau + k = \mu$$

μ is the *total linear attenuation coefficient*

$$\sigma = \sigma_R + \sigma_C \quad (4.2)$$

σ_R is the Rayleigh scattering, σ_C the Compton scattering, τ the photoeffect and k pair production contributions to the total attenuation coefficient.

4.1.1 Mean Free Path

Mean free path is the average distance that a photon can travel in an attenuator without having any kind of interaction. This is analogous to the 'mean life' in radioactive decay process. *The mean free path or mean path length or relaxation length is the reciprocal of the total linear attenuation coefficient (μ^{-1} cm).* The attenuator thickness may be expressed in units of mean free path. Hence a thickness x may be expressed as $x/\mu^{-1} = \mu x$ which is dimensionless.

4.2 Mass Attenuation Coefficient (μ/ρ)

It is the ratio of the corresponding linear attenuation coefficient to the density of the attenuator in g/cm^3 . *Mass attenuation coefficients are independent of the physical state of the attenuator.* Because it is possible to express various interactions in terms of cross section per atom, we may obtain the mass attenuation by multiplying these quantities by the number of atoms per cm^3 of the given substance. So mass attenuation coefficients are more fundamental and important than linear attenuation coefficients,

4.3 Total photon attenuation cross section

The linear attenuation coefficient μ gives the fractional reduction of the incident beam intensity on passing through a thin layer of thickness dx of an absorbing medium.

$$-\frac{dI}{I} = \mu dx \quad (4.3)$$

On integrating this relation, we get the equation (1.1) for transmitted intensity in the case of a homogeneous medium

$$I = I_0 e^{-\mu x}$$

$$\ln \frac{I_o}{I} = \mu x \quad (4.4)$$

$$\text{ie.} \quad \mu = x^{-1} \ln \left(\frac{I_o}{I} \right)$$

$$\frac{\mu}{\rho} = \frac{\ln(I_o/I)}{\rho x} \quad (4.5)$$

$$\text{ie.} \quad \mu t = \rho \ln \left(\frac{I_o}{I} \right) \quad (4.6)$$

where $t = \rho x$ is the sample thickness in g/cm^2 μ is in cm^2/g , therefore μt is dimensionless. The mass attenuation coefficient μ/ρ may be written as

$$\frac{\mu}{\rho} = \frac{N_A}{A} \sigma_{total} \quad (4.7)$$

$$\text{or} \quad \sigma_{total} = \frac{A}{N_A} \frac{\mu}{\rho} \quad (4.8)$$

$$\text{ie.} \quad \sigma_{total} = \frac{A}{N_A} \frac{\ln(I_o/I)}{t} \quad (4.9)$$

σ_{total} is the total photon attenuation cross section, A is the atomic weight of the attenuator, and N_A is the Avogadro constant. The procedure involved in the measurement of total attenuation cross section is given in section 4.4

4.4 Experimental procedure

The good geometry setup shown in Figure 3.1 was used to perform the transmission experiment. The gamma ray source (one from Table 3.3) was housed in the source position S as shown in the Figure 3.1. Keeping an empty plastic container, which is identical with that used to fill the sample, in the path of the collimated beam of gamma rays the transmitted spectrum was recorded [Figure: (4.1 to 4.3)] and the intensity I_o is measured. The experiment was repeated after replacing the empty container with the container filled with the sample (compound). The corresponding transmitted intensity I is measured. Both of these measurements were

Inner spectrum — with sample
Outer spectrum — without sample

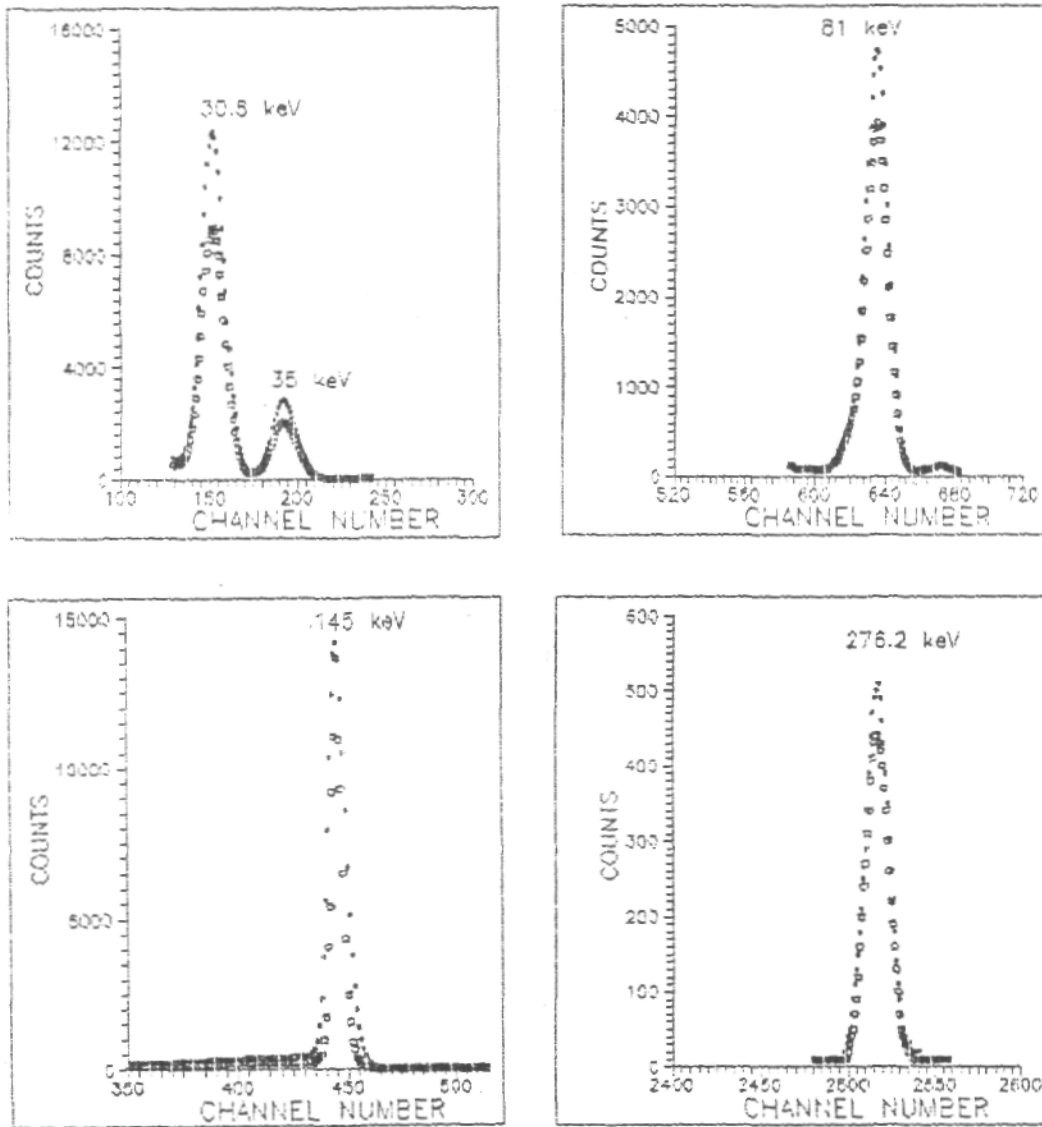


Figure 4.1: Spectrum recorded for various energy peaks

Inner spectrum — with sample
Outer spectrum — without sample

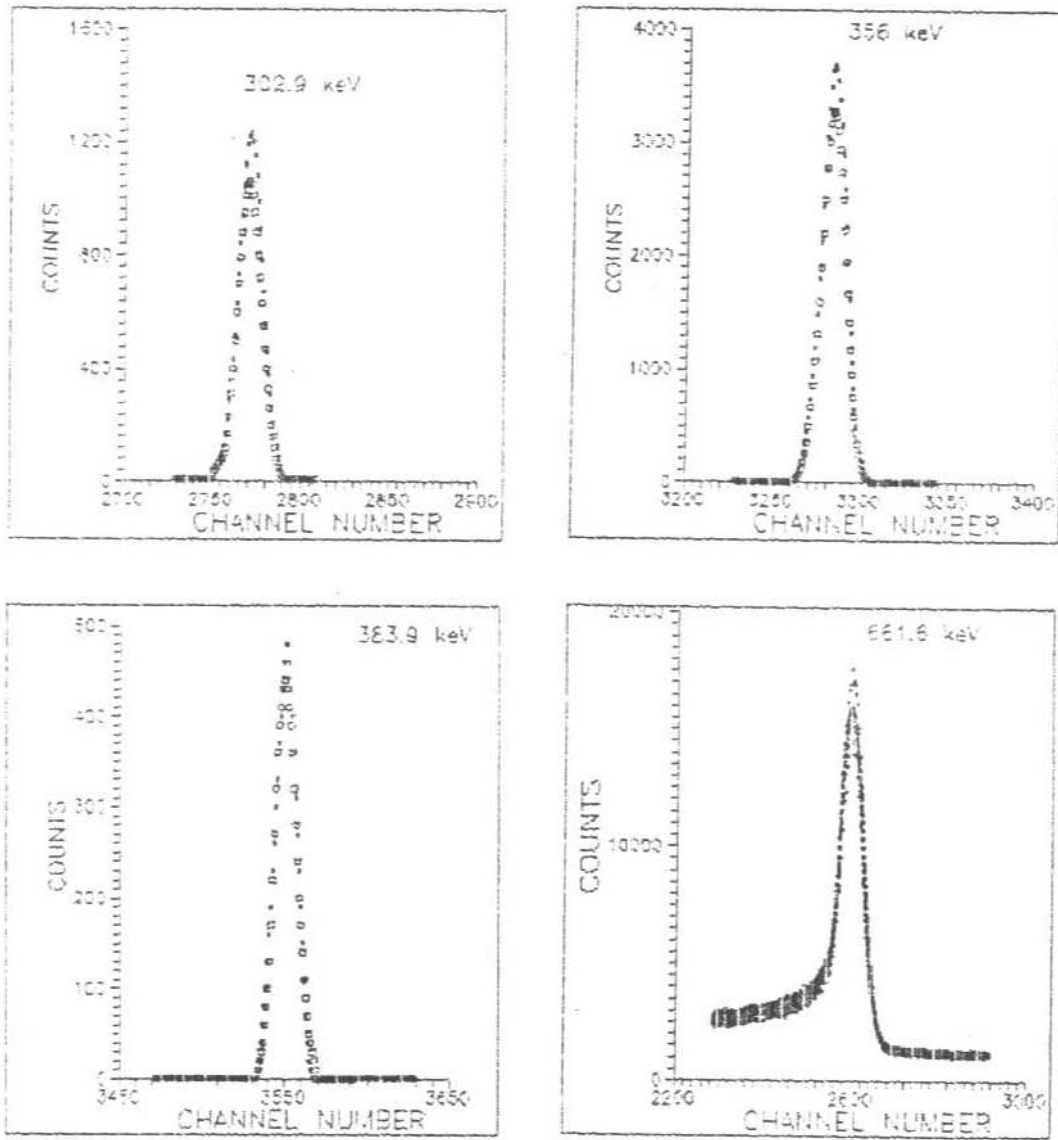


Figure 4.2: Spectrum recorded for various energy peaks

Inner spectrum — with sample
Outer spectrum — without sample

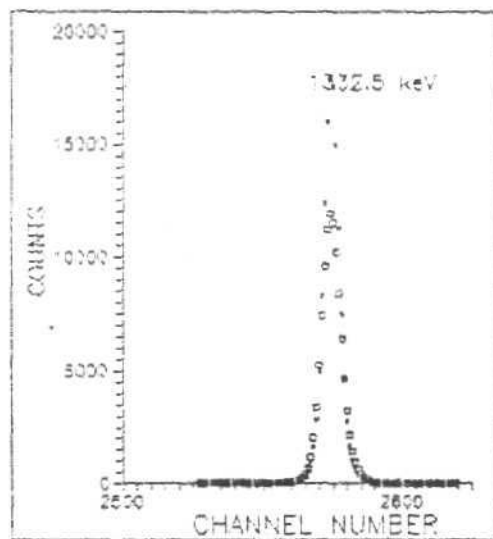
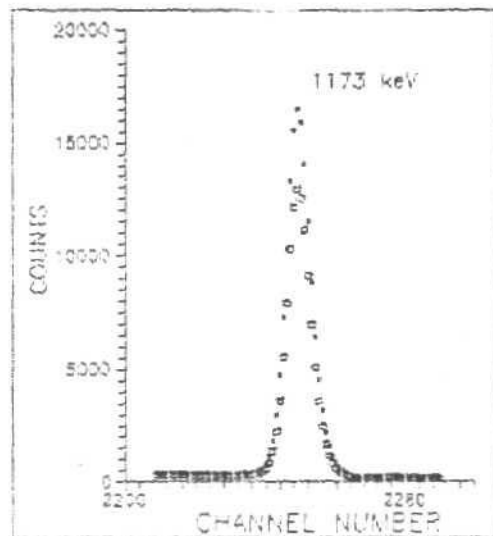


Figure 4.3: Spectrum recorded for various energy peaks

repeated a number of times, under identical conditions, for the same compound with different μt values. The counting time was suitably selected so that in each case, at least 10^5 to 10^6 counts were recorded at the photopeak of the recorded spectrum (Figure 4.1 to 4.3). So the error due to counting statistics was below 0.3 %.

The experimental procedure for the measurement of total attenuation cross sections followed by most of the investigators includes the determination of the total number of counts under the entire photopeak (Shivashankara Rao et al. 1973, Parthasaradhi and Hansen 1974, Reddy et al. 1976, Premchand et al. 1976, Radhakrishna Murty et al. 1977, Rajendraprasad 1978 and 1980) or select a few channels around the centroid of the photopeak (Gopal and Sanjeevaiah 1973, Ramakrishna Gowda and Sanjeevaiah 1974, Ramakrishna Gowda et al. 1976, Putaswamy 1980, Umesh et al. 1981 & 1983) and the corresponding counts are used for the calculation of total attenuation cross section.

In the current study two-two channels symmetrically located on either side of the centroid of the photopeak were selected and their counts added. This procedure was extended to cover the entire photopeak. In each step the unattenuated and attenuated intensities I_o and I were recorded. Corresponding to each set of values so obtained, the total attenuation cross section σ_{total} was calculated using equation (4.9)

$$\sigma_{total} = \frac{A}{N_A t} \ln(I_o/I) \quad (4.10)$$

where N_A is the Avogadro constant, A the molecular weight and t the sample thickness (g/cm^2)

$$\sigma_{total} = \frac{A}{0.6025t} \ln(I_o/I) \quad \text{barn/molecule.} \quad (4.11)$$

Error in the measurement, due to multiple scattering effects, was minimum because thickness of the samples of amino acid and sugar compounds used in the present investigation were optimized such that $0.1 < \mu t < 0.4$

(section 3.3 and 3.4), and a hyper pure Germanium detector of good resolution was used to detect the transmitted photons. However an extrapolation technique was used to check this minimum effects of multiple scattering (Puttaswamy 1979, 1980). In this method the total attenuation cross sections obtained were plotted against channel number around the centroid of the photopeak. The resultant curve was extrapolated to the centroid which has eliminated the multiple scattering effects.

The procedure described above, was repeated several times for each compound. In each case, the extrapolated value was determined. Average of these values, σ_{total} , was obtained for each compound at the given energy. The entire experiment was repeated for photon energies 30.8 (K_{α} -Xray), 35 (K_{β} -Xray), 81, 276.4, 302.9, 356 & 383.9 keV from ^{133}Ba , 145 keV from ^{141}Ce , 661.6 keV from ^{137}Cs and 1173 & 1332.5 keV from ^{60}Co radio active sources. The average values of total attenuation cross sections σ_{total} obtained are tabulated in Chapter 5 (Table 5.4 to 5.25).

4.5 Experimental errors

4.5.1 Secondary radiation and its effects

When gamma rays pass through matter the probability for different interactions with atoms depends on energy of the incident photons and atomic number of the attenuator. In the three major types of interaction, only by photoeffect does the primary photon disappear. A photoelectron is emitted from the atom during this process. A photon undergoing Compton collision continues to exist with lesser energy. A Compton electron is emitted during this process. When a photon is absorbed during pair production, the annihilation radiation appears. The photons along with the electrons and positrons produced as a result of interaction of the primary photons together with the characteristic X-rays and auger electrons

produced following the filling-up of the inner shell vacancies are called the secondary radiations. The secondary radiation in turn interacts with the sample atoms to produce, still energy degraded tertiary radiations which includes Bremsstrahlung radiation produced by electrons (section 4.5.6). The tertiary radiation in turn produces the next generation of further energy degraded radiation and so on. This phenomenon is called *the multiple scattering of gamma radiation*. It is a usual practice to call all these radiations, except the primary one, together as *secondary radiations*.

To estimate the multiple scattering effects we should know the nature of the secondary radiations. In the study of secondary radiations, their nature and distribution with respect to energy and direction of motion at a point in the medium, it is useful to consider the following quantities:-

The Distribution Function

Let us consider unit area at a given point of space such that the normal to it makes the angle $\vec{\Omega}(\theta, \phi)$ with the direction from the point source to the given point. The function $N(\vec{r}, \vec{\Omega}, E)$ is called the distribution function of the flux of photons with respect to direction and energy, if $N(\vec{r}, \vec{\Omega}, E)$ is the number of quanta passing in unit time through the given unit area with an energy lying between E and $E + dE$, through an element of solid angle $d\vec{\Omega}$ about the direction $\vec{\Omega}$. It is having dimensions $\text{cm}^{-2} \text{sec}^{-1} \text{erg}^{-1} \text{steradian}^{-1}$ in the cgs system.

The energy flux which is important in theoretical calculations of energy dose in biological systems, is defined as

$$I(\vec{r}, \vec{\Omega}, E) = E \cdot N(\vec{r}, \vec{\Omega}, E) \quad (4.12)$$

Its unit is $\text{cm}^{-2} \text{sec}^{-1} \text{steradian}^{-1}$. $I(\vec{r}, \vec{\Omega}, E)d\vec{\Omega} dE$ represents the amount of energy carried by the quanta, the energy of which lies in the interval dE , and the direction of motion in the interval $d\vec{\Omega}$, in unit time through that unit area whose normal lies in the direction of $d\vec{\Omega}$

Flux and Current

The quantity

$$I_o(\vec{r}, E) = \int_{4\pi} I(\vec{r}, \vec{\Omega}, E) d\Omega \quad (4.13)$$

is called the energy flux of the gamma ray. The flux of photons $N_o(\vec{r}, E)$ may be defined in a similar manner. The units of these quantities are $\text{cm}^{-2} \text{s}^{-1}$ and $\text{cm}^{-2} \text{s}^{-1} \text{erg}^{-1}$ respectively.

Build-up Factors

When transmission experiment is performed, the detector may record primary as well as some scattered photons. In conditions of good geometry the detector reading is concerned only with primary photons. The results of experiments and calculations may be represented as ratio of detector readings under conditions of *bad geometry* to those under conditions of *good geometry*. This ratio is called the build up factor. So build-up factor is $B = (\text{effect produced by all the photons})/(\text{effect produced by primary photons})$. Depending on the nature of the detecting system build up factor may be defined as follows.

1. If the instrument records number of photons, build-up factor

$$B_N(\vec{r}) = \frac{\int N_o(\vec{r}, E) dE}{\int N_o^o(\vec{r}, E) dE} \quad (4.14)$$

The numerator represents flux of photons of all energies and the denominator represents flux of only primary photons.

2. If the instrument records energy flux, build-up factor for energy flux is

$$B_E(\vec{r}) = \frac{\int I_o(\vec{r}, E) dE}{\int I_o^o(\vec{r}, E) dE} \quad (4.15)$$

The numerator represents the amount of energy carried by all the photons and the denominator represents the energy flux due to primary photons reaching the detector.

3. Build-up factor for the energy dose

$$B_D(\vec{r}) = \frac{\int \mu'_a(E) I_o(\vec{r}, E) dE}{\int \mu'_a(E) I_o^o(\vec{r}, E) dE} \quad (4.16)$$

$\mu'_a(E)$ is the linear energy absorption coefficient for photons with energy E for air at NTP.

The numerator represents the energy lost in unit time in unit volume of air by the primary and secondary photons. The denominator represents the corresponding quantity only for primary photons.

The integration in the above equations are to be performed within limits 0 to E_{max} . From the above equations one may observe that B is always > unity.

Apart from multiple scattering, there are several possible errors, in the measured values of total attenuation cross sections. They are due to:-

1. Counting statistics
2. Small angle scattering contribution
3. Non-uniformity of the sample compounds
4. Sample impurity
5. Photon dose building effects
6. Dead time of the counting instrument
7. Pulse pile-up effects.

4.5.2 Counting statistics

In the transmission experiment the counting time was selected such that at least 105 to 106 counts were recorded under the photopeak. So the statistical uncertainty in the counts was < 0.3%.

4.5.3 Small angle scattering contribution

In the experimental setup a distance of about 50 cm was provided between the sample and the detector. This corresponds to a maximum angle of acceptance 31 minutes for the scattered photons. According to the theoretical estimates, the contribution of coherent as well as incoherent scattering at such small angles to the measured cross sections at the energies of interest is negligibly small (0.6 to 0.3% at 1 MeV, at lower energies this contribution should still decrease). Hence no small angle scattering corrections were applied to the measured cross sections.

4.5.4 Non-uniformity of the sample

Non-uniformity of the samples was checked by exposing different parts of the samples to the incident beam. Any discrepancy in the attenuated, transmitted, intensity in each case was found to be within counting statistics.

Carter et al. (1967) gives a relation for calculation of the error in mass absorption coefficient μ/ρ due to absorber non-uniformity as

$$(\Delta\mu/\rho) = \frac{1 - \rho/a \sinh(a\mu/\rho)}{\ln \bar{R}} \quad (4.17)$$

where $(\Delta\mu/\rho)$ is the error in mass absorption coefficient, \bar{R} is the mean transmission ratio for obtaining the values μ/ρ and a is the maximum deviation of absorber thickness from the mean thickness (gm cm^{-2}) and a/ρ is the thickness in cm. In the present investigation the uncertainty in the mass per unit area was less than 0.005%. So the error in the cross section due to non-uniformity of the sample calculated using the above equation was found to be less than 0.005 % for all the samples at the energies of interest.

4.5.5 Sample impurity

The error due to sample impurity can be high only when large percentage of high Z impurities are present in the sample. In all the compounds used in this investigation, the content of high Z impurities was less than 0.005 %. Hence no sample impurity corrections were applied to the measured cross sections.

4.5.6 Photon dose build up effects

When gamma rays pass through matter the photons may be absorbed or scattered during interaction with atoms, depending on atomic number Z of the atom, the incident photon energy and the absorber thickness. Following photon absorption, electrons are ejected from $K, L, M...$ energy levels, creating vacancies in these levels. When these electron vacancies are filled by electrons in the upper energy levels, characteristic X-rays are produced. The photoelectrons, Compton electrons, auger electrons etc. produced inside the absorber may suffer inelastic collisions in the Coulomb field of the atomic nuclei, resulting the emission of bremsstrahlung radiations. In addition to the above processes, the multiple scattering of photons inside the sample produces energy degraded photons. All these complex secondary processes create a net photon dose build up inside the sample material. According to quantum-mechanical theory of bremsstrahlung, the possibility of their emission, and intensity in compounds used in the present study, which contain low Z elements are negligibly small, and in most of the cases the electron suffers elastic collision in the Coulomb field of the nucleus. For a given incident energy and compound the intensity of secondary radiations reaching the detector, in the given setup, is proportional to the sample thickness. The sample thickness in this study was optimized (section 3-3 and 3.4) which has effectively reduced the effects of secondary radiations. Since an optimum count rate, counting time and a detector of

good resolution were employed, it is expected that the photon dose build up effects were negligible in the current study. Still, the error due to multiple scattering effects, if present any, have been corrected by the extrapolation technique.

4.5.7 Dead time of the detecting system

In the multichannel analyzers used, there was a built in provision for dead time correction.

4.5.8 Pulse pile up effects

The optimum count rate and counting time (section 4.5.2) employed in the study have kept the pulse pile up effects to a minimum, which is negligible.

The above description shows that the overall error in the measured cross sections was less than 2 % for all the compounds and energies of interest.

It may be argued that in the case of ^{133}Ba radioactive source, since the incident beam consists of photons of various energies, corrections must be applied for the *contribution from higher energy photons* (due to Compton scattering in the detector and the target materials) to the low energy photon intensities. However, in the present study, *since a detector of high resolution was used together with an extrapolation technique which corrects for the effects of multiple scattering*, it was felt that any such effect would be within the range of experimental errors. Hence no such corrections were applied to the measured data.

The experiment was repeated with different samples of the same compound, with different μt values ranging from 0.1 to 0.4 yielded concordant values of total attenuation cross sections. It shows the existence of a unique value for μ in the thickness range used in the investigation.