

**THESIS**  
**ON**  
**PROBLEMS ON VORTEX MOTION**  
**together with**  
**SUPPORTING PAPERS**  
**submitted to**  
**THE UNIVERSITY OF MYSORE**  
  
**for the award of the degree of**  
**DOCTOR OF PHILOSOPHY**  
  
**by**  
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## INTRODUCTION

The Thesis on "Problems on Vortex Motion" submitted herein, consists essentially of two distinct types of problems. The first, consisting of two published papers relating to four rectilinear vortex filaments, is the principal contribution to the thesis. The recent work of Synge on three vortices is extended to four, and the problem of finding all possible types of fixed configurations of four vortices is completely solved. The general pattern of all possible motions is discussed for the particular case where the configuration is initially a parallelogram and the vortices at the ends of diagonals are of equal strength. The results obtained in the above two papers contain extensions of the work previously done by Grobli, Morton and others.

The second type of problems of an existential nature relate to the notion of vorticity as such. The four papers of this category included in the second part, give answers to particular questions posed about the nature of vorticity in different kinds of fluids. Thus, the first paper investigates conditions under which, in a viscous compressible fluid, there is no decay of vorticity in spite of viscosity and in which certain flows possess properties similar to those of inviscid fluids. The second paper which relates to viscous fluids obtains an expression for the velocity of such a fluid within a fixed container in terms of the expansion and vorticity. The third paper extends a known result about vorticity transport to the case of steady compressible flow of a fluid of constant kinematic viscosity. The fourth paper whose interest is purely mathematical deals with the notion of vector-lines in a general Riemannian  $V_n$  and determines the condition for the permanence of vector-lines in such a space.

The thesis consisting of the above two parts is supplemented by ten other original papers in support of the application for the degree. Of these ten papers three relate to Hydrodynamics, and are therefore indirectly connected though not intimately, with the subject matter of the thesis. The first two of these three papers investigate axisymmetric flows with a common flow pattern and study a class of such flows of viscous liquids. The third paper extends a wellknown result relating to the Joukowski aerofoil profile to a general aerofoil profile and to the S-form profile of R.v.Mises.

The remaining seven papers consist of three on applied mathematics and four on pure mathematics. The former consist of one on vibrations, one on heat conduction and another on elasticity. The latter consist of two papers on trigonometric summations, one on a method of evaluating the Dirichlet's Integral using the Laplace Transform and another on positive complex matrices.

Of the papers comprising the thesis, the principal ones relating to four vortex filaments have been published in the Proceedings of the Indian Academy of Sciences, vols. 34(1951) and 38(1953). The first paper of the second part of the thesis is in the course of publication in the Journal of the Indian Institute of Science. The remaining papers are submitted here for the first time.

Of the supporting papers, the one on "A class of Axisymmetric Motions of Viscous Liquids" is to be published in Math. Zeitschrift, the one on vibrations of composite strings (joint work with Dr. B.S. Rama Krishna) is to be published in the American Journal of Physics, and the one on heat conduction in an infinite octantal solid, in the Journal of Mathematics and Physics, Massachusetts Institute of Technology. The paper on Dirichlet's Integral

has been accepted for publication in the American Mathematical Monthly and the papers on the Area and Equation of the General Aerofoil Section and A Property of Complex Matrices have been submitted for publication in the same monthly. The papers on trigonometric summations (joint work with Dr. B.S. Rama Krishna) have been published, one in Nature, vol.171 (1953) and the other in the Proceedings of the Indian Academy of Sciences, vol.36(1952). The remaining papers are submitted here for the first time.

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The work started in April 1951, when I was in the Mysore University, was continued after I joined the Indian Institute of Science, Bangalore, with the help, support and encouragement of Professor N.S. Govinda Rao, B.E., M.I.E., F.A.Sc. To him I am very much indebted. My sincere thanks are due to the joint author for the kind collaboration in some problems. I take this opportunity to thank the authorities of the Indian Institute of Science, Bangalore, for enabling me to do this work.

THESIS

PART A

FOUR RECTILINEAR VORTICES

- (1) Fixed Configurations of Four Rectilinear Vortex Filaments
- (2) The Motion of Four Rectilinear Vortex Filaments

PART B

MISCELLANEOUS PAPERS ON VORTICITY

- (1) On a Class of Viscous Compressible Flows
- (2) The Velocity of a Rotational Viscous Flow Inside a Fixed Container
- (3) Transport of Vorticity in a Compressible Flow
- (4) On the Permanence of Vector-Lines in a Riemannian  $V_n$

SUPPORTING PAPERS

PART A

PAPERS ON HYDRODYNAMICS

- (1) Axisymmetric Fluid Motions with a Common Flow Pattern
- (2) A Class of Axisymmetric Motions of Viscous Liquids
- (3) The Area and the Equation of the General Aerofoil Section

PART B

- (1) Transverse Vibrations of Composite Strings
- (2) A Problem in Heat Conduction and the Product of Three Error Functions
- (3) The Torsional Rigidity of a Bar with Sectorial Cross-Section

- (4) A Trigonometric Series Used in Physical Problems
- (5) Certain Trigonometric Summations
- (6) On the Evaluation of Dirichlet's Integral
- (7) A Property of Complex Matrices.