

STUDIES IN STABILITY THEORY OF DIFFERENTIAL EQUATIONS

Thesis submitted to
The University of Mysore for the award of the Degree of
DOCTOR OF PHILOSOPHY

by

N. B. BADRINARAYAN
Lecturer in Mathematics
Regional College of Education, Mysore 570 006

MYSORE UNIVERSITY LIBRARY.
MYSORE-6.

Under the Guidance of

Dr. B. SIDDAPPA Ph.D (Dublin)
Professor and Head
Department of Studies in Mathematics
Gulbarga University
Gulbarga 585 105 (India)

OCTOBER 1981

**DEDICATED TO THE MEMORY OF
MY MOTHER**

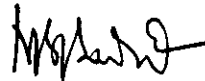
**MYSORE UNIVERSITY LIBRARY
MYSORE-6.**

DECLARATION

I hereby declare that the contents of this thesis entitled ' STUDIES IN STABILITY THEORY OF DIFFERENTIAL EQUATIONS ' are the results of my research carried out under the Guidance of Dr.B.Siddappa, Ph.D.(Dublin), Professor and Head in Department of Studies in Mathematics, Gulbarga University, Gulbarga 585 105 (India).

The material in this thesis has not been submitted by me earlier for the award of any degree, diploma, associateship, fellowship or any other similar title.

Date: 5/10/81



M.B. Badrinarayan
Lecturer in Mathematics
Regional College of
Education, Mysore-570 006

MYSORE UNIVERSITY LIBRARY
MYSORE-6.

C E R T I F I C A T E

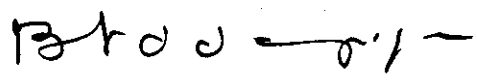
The research work done by
Sri Nanjangud Sheema Rao Badrinarayana
entitled

"STUDIES IN STABILITY THEORY OF DIFFERENTIAL EQUATIONS"
was carried out and completed under my guidance. The
candidate has completed the research work and is hereby
permitted to submit the thesis to the University of
Mysore for the award of the Ph.D., degree.

The material presented in this thesis has not
been used by the candidate for the award of any degree,
diploma, associateship, fellowship or any other similar
title.

Date:

5.10.81.


(B. SIDDAPPA, Ph.D. Dublin)
Professor & Head,
Department of Studies in
Mathematics
Gulbarga University,
Gulbarga-585 105 (India)

ACKNOWLEDGEMENT

I thankfully acknowledge that the problems of research embodied in this thesis entitled "STUDIES IN STABILITY THEORY OF DIFFERENTIAL EQUATIONS" were suggested by Dr.A.A.Kayande, Nagpur.

My best regards are to my guide Dr.B.Siddappa,Ph.D., (Dublin), Professor and Head in Department of Studies in Mathematics,Gulbarga University, Gulbarga, who gave me his helping hand at a crucial moment of my work when help was needed most. He gave me confidence, spent much of his valuable time with me discussing my problems and giving precious suggestions. But for his timely help, I very much doubt whether I would have completed my work. I thank him beyond words and am extremely grateful to him.

My thanks are due to my well wishers within and out side my college, whose keen interest and encouragement meant enough fuel to keep me going the whole hog of research. Their constant enquiries and very considered suggestions helped me greatly. I thank the college and the NCERT for the contingency grant sanctioned for the work.

Mr.M.S.Ramaswamy, has typedent the thesis in the present form, very punctually and my thanks are due to him.

Despite human effort, I believe that the Divine Grace is a must for the completion of any work. This came to me through the Blessings of the Lord Raghavendra of Mantralaya. My infinite salutations are to the Lord.

Date: 5/10/81


(H.B. BADRINARAYAN)
Lecturer in Mathematics

CONTENTS

<u>Chapter/Section</u>	<u>Title</u>	<u>Page No.</u>
	General Introduction	1 - viii
I	Stability of General Dynamical Systems.	
1	Introduction	1
2	Preliminaries	4
3	Comparison Theorems	10
4	Sufficiency Conditions for Stability	15
5	Converse Theorems	31
6	Conditional Invariancy of Set B with respect to Set A for a GDS in E	40
7	Conditional (or relative) Stability of a compact set A with respect to a GDS in E	56
8	Converse Theorems for the existence of Vector Lyapunov functions for conditional stability	66
II	Stability of Conditional Invariant Sets of Control Systems.	
1	Introduction	74
2	Sufficiency conditions for stability of a conditional Invariant set with respect to a control system	76
3	Stability behaviour of conditional Invariant Set B with respect to A for the Control System.	91

<u>Chapter/Section</u>	<u>Title</u>	<u>Page No.</u>
4	Theorems on asymptotic stability of conditional invariant set B relative to A for the system (2.1)	100
5	Control System as a Perturbed System	112
6	Optimisation Problem	121
III	Stability of a Set B with respect to a Set A, for a Differential System.	
1	Introduction	124
2	Stability concepts of a set B relative to a set A for a differential system	126
3	Sufficiency conditions for the stability of a set B relative to a set A for the system	133
4	Extension of comparison technique to the study of stability of set B relative to set A for the system (2.1)	140
5	Existence of Lyapunov functions under the assumptions of stability of set B relative to set A for the system (2.1)	147
6	Some results on the Stability of B relative to A	153
7	Extension of conditional stability to the stability properties of set B relative to A	157
IV	General Stability-like concepts for differential and control system	
1	Introduction	164
2	General sufficiency conditions for stability	166
3	Theorems on the existence of Liapunov functions	182
4	Control systems	188
	Bibliography	193