SUMMARY

Distributions of extreme values are useful for solving some of the important problems arising in the field of agricultural production with water-management engineering, viz., extreme meteorological phenomena like, temperature, rainfall, runoff etc; breaking strength of materials; floods and droughts. In country like India, these phenomena occur frequently at regular intervals but in a very erratic fashion. More or less, similar problems arise from a sudden scarcity or surplus of runoff or rainfall, which need a special attention and thoughtful study to arrive at immediate solutions. The present investigation is an attempt to develop suitable techniques for dealing with data relating to such phenomena.

Some useful results from extreme order statistics are organised in chapter I. These results are regarding (a) the asymptotic distribution functions of extreme order statistics due to Fisher and Tippett (1928) and the conditions for their domain of attractions due to Gnedenko (1943). (b) the bivariate asymptotic distribution functions due to Gumbel and Mustafi (1967), Sibuya (1960), Tiago de Oliveira (1959) and Geffroy (1958) and the conditions for the independence of marginals. (c) the statistical tools of extreme order statistics applicable to applied problems.

A brief review concerning the asymptotic theory of extreme order statistics, estimation of the parameters of asymptotic distribution functions and the applications of the theory to various fields is included in chapter II.

Chapter III is devoted to the estimation of the parameters of type I distribution function. The estimates, available so far are based on the use of maximum order statistics only. Naturally these estimates can be made more efficient by using additional information. To achieve this end, the estimates are derived with the knowledge of m maxima of samples. The covariance matrix of such estimates has also been given. Similarly, the estimates and their covariance matrix are obtained with the help of joint distribution function of m maximum observation from several samples.

The study revealed that the estimate of location parameter is more efficient when one uses the second maximum. However, the efficiency of the estimate of scale parameter increases continuously if second, third, fourth and fifth maximum values from several samples are used. The rate of increase in efficiency is highest when the second maximum observations rather than the maximum observations are used. However, taking lower ranked observations, efficiency decreases and ultimately reaches zero.

In chapter IV, a study of location parameter (λ), scale parameter (δ) and shape parameter (k) of Frechet distribution is made. Using the knowledge of m-th maximum, the estimates are obtained by the usual principle of maximum likelihood estimation. The estimates and their covariance matrices are calculated for different values of m = 1,2,3,4,5 and the shape parameter $k = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, 4$. For convenience, to estimate the parameters λ is assumed to be known. Subsequently the results have been generalised taking λ as well unknown. Similar results are derived with the joint knowledge of m maximum observations.

In chapter V and VI we deal with the estimation of parameters of bivariate distribution functions with Gumbel and Frechet distributions as marginals. It is observed that the method of maximum likelihood does not lead to the simultaneous estimation of all the parameters of bivariate distributions. To over come this difficulty it is suggested that the estimates of the scale, location and shape parameters can be obtained by observations on component variables and consequently one can estimate the remaining dependence parameter assuming the other parameters to be known constants. The estimates along with their covariance matrix are derived when the marginals are I and II asymptotes of first and second largest values. For convenience, to estimate the parameter of distribution function with Frechet marginals λ is assumed to be known. Subsequently the result have been generalised taking λ as well unknown.

The applications of extreme value theory to the flood analysis are discussed in chapter VII. To introduce the topic, the theoretical results of Todorovic and his coworkers are given. These results are extended for two dimensional variables. It has been shown that the distribution function of the number of exceedances is a bivariate poisson distribution with time dependent intensity function. The expected number of exceedances in a fixed time interval are expressed by a finite Fourier series. The bivariate distribution functions for supremum and infimum of the magnitude of exceedances in two variables have been obtained under the assumption that the number of exceedances and their magnitude are independent.

The magnitude of exceedances is assumed to be exponentially distributed. The application of univariate work has been illustrated with the data of river Narmada at two stations namely Mortakka and Garudeshwar separately. The parameter α_1 and α_2 of univariate exponential distribution are estimated using the method of maximum likelihood. The application of derived bivariate theory is illustrated with the help of the same data. The dependence parameter of bivariate distribution function is estimated by relating it to the medial correlation coefficient. Finally, the return period in bivariate case is derived. The goodness of fit of theoretical and observed results was demonstrated graphically and is judged to be satisfactory.

Limitation of present investigations and open problems:

The present investigations mainly center around the Gumbel and Frechet distributions where the results for two dimensional variables have been derived on the basis of the findings of Todorovic (1970) .

All the results of chapter III and IV concerning the estimation of Gumbel and Frechet distribution clearly reveal that these results may further be extended to Weibull distribution also. This is because Weibull distribution plays an important role in the problems of breaking strength of materials and fatique failure which are analogons to the problem of these studies. Further more, since in these studies due to the continuous nature of 'flood level', we have exclusively concentrated on the derivation of continuous variables and the results achieved don't have any blanks, but the study of discrete variables also draws our attention. This is because, in practice (pertaining to industrial applications), the exceedances of discrete variables may be of utmost importance and therefore need further investigations. Then efforts may be made to investigate the pattern concerning the exceedances of $X(t)$, while taking it as discrete variable .