CHAPTER

9

CHAPTER 9

DISCUSSION OF BIAS, MEAN SQUARE ERROR AND RELATIVE EFFICIENCY.

In this chapter we shall discuss the results of bias, mean square error and relative efficiency of the true error estimate, V, obtained by the proposed procedure as defined in Chapter 8. The discussion which follows is based on the theoretical results obtained in the preceding chapter and computational results assembled in Appendix B. We shall be particularly interested in comparing the mean square error of V with that of an unbiased estimate V_3 of the true error and in making recommendations for the use of proposed estimation procedure.

The bias and mean square error of the estimate V are functions of 7 parameters : three degrees of freedom n_1, n_2, n_3; two levels of significance α_1 and α_2 ; and two parameters ψ_{12} and ψ_{23}. Out of these seven parameters n_i's are fixed by the experiment, ψ_{12} and ψ_{23} are nuisance parameters and hence none of these is at the choice of the experimenter. Our discussion for the use of the proposed estimation procedure will therefore be confined to a suitable choice of the two levels of significance α_1 and α_2. In order to simplify calculations we have taken $\alpha_1 = \alpha_2 = \alpha$ (say).

To illustrate the bias and mean square error of V we have considered two sets of degrees of freedom n_1, n_2 and n_3. In one, we have taken the true error degrees of freedom n_3 small and in the other case n_3 has been taken large. Tables 1 - 6 assembled in Appendix B show bias, mean square error and relative efficiency of V to V_3 for $\alpha = 0, 0.05, 0.25, 1.0$. corresponding to different values of ψ_{12} and ψ_{23}. All these calculations have been made on IBM 7044 computer installed at I.I.T. Computer Centre, Kanpur and we acknowledge with thanks the service rendered by the centre.

9.1. Discussion On Bias

In this section we discuss the results of bias. Tables 1 and 2 give results of bias expressed as a fraction of σ_3^2. These tables indicate that for $\alpha = 0$, the numerical value of bias decreases as ψ_{12} and/or ψ_{23} increases and becomes zero at $\psi_{12} = \psi_{23} = 1$. For $\alpha = 1$, the bias is always zero; as this corresponds to using the estimate V_3. For $\alpha \neq 1.0$, we observe that the absolute value of bias decreases as we increase α for $\rho_{12} \leq 0.6$. If we exclude these extreme cases $\alpha = 0$ and $\alpha = 1$, we observe that for $\rho_{12} = \rho_{23} = 1$, the bias increases as α is increased and is greater than $\alpha(\alpha - 2)$ in magnitude and confirms the result 8.3.2. For a fixed value of ρ_{12} and $\alpha = 0.05$, the absolute bias increases and then it decreases as we increase ρ_{23}. The same is true for ρ_{23}. It is also observed from these tables that the magnitude of bias expressed as a fraction of σ_3^2, is greater when n_3 is small than when n_3 is large. Any slight deviation in Computed Values may be due to the computational approximations.

9.2. Discussion of Mean Square error

Next, we discuss the results of mean square error. Tables 3 and 4 in Appendix B show the mean square error of V as a fraction of σ_3^4. When n_3 is small and we use the estimate based on preliminary tests of significance, we observe that the estimate has a mean square error which is in general, smaller than the variance of the unbiased estimate V_3 for all values of the nuisance parameters. When n_3 is large, this is true provided $\rho_{12} \geq 0.6$. For ρ_{12} fixed, the mean square error of V, in general, decreases as we increase ρ_{23}. There is no such tendency howsoever, when we fix ρ_{23}.

9.3. Relative Efficiency

Now, we discuss the results of relative efficiency of V to V_3. We denote the relative efficiency of V to V_3 by R.E.(V) and define it by $\dfrac{2\ \sigma_3^4/n_3}{MSE(V)} \times 100$, where $MSE(V)$ denotes the mean square error of V. The computed values of R.E. have been given in tables 5 and 6 of Appendix B. From these tables we see that for n_3 small and $\varphi_{23} > 0.4$ V is more efficient than V_3 for $\alpha = 0.05$ or $\alpha = 0.25$ but the magnitude of the relative efficiency is more when $\alpha = 0.05$ than for $\alpha = 0.25$. When we take $\alpha = 0$, V is always more efficient and has the maximum relative efficiency. When n_3 is large and we take $\alpha = 0.25$, V is more efficient or almost equally efficient as V_3 for all values of φ_{23}.

9.4. Recommendations

From the previous discussions we note that an indiscriminate use of 'always pool' estimate (i.e. $\alpha = 0$) is not good as it is highly biased. Similarly, an indiscriminate use of 'never pool' estimate is also not good as it results in a variance which is larger than the mean square error of the estimate V based on the preliminary testing.

When only large values of φ_{23} are envisaged as a possibility by the experimenter, it is suggested that he should use $\alpha = 0.05$, but for small values of φ_{23}, he is advised to use $\alpha = 0.25$ or even $\alpha = 0.50$. φ_{12} does not appear to have an appreciable effect on the mean square error of V and the relative efficiency of V to V_3.