

SOME CONTRIBUTIONS
TO
DYNAMIC PROGRAMMING

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G.V. Shenoy

P R E F A C E

Major part of the material presented in this thesis covers the following research papers which have been published, accepted for publication or presented at the conferences :

- (i) "Production scheduling and employment smoothing problem under uncertain demand", *The Logistic Review*, Vol.6(27), 1970.
- (ii) "A solution to the optimal production scheduling problem", presented at the Third Annual Convention of O.R. Society of India, held at Hyderabad, during December, 1970.
- (iii) "On dynamic programming and risk functions", to appear in the *Cahiers du Centre d' Etudes de Recherche Operationnelle*, Belgium, Vol. 13, 1971.
- (iv) "An optimization procedure using dynamic programming technique", accepted for publication in the *Journal of the Association Francaise Pour La Cybernetique Economique et Technique*, France.
- (v) "On the warehousing problem", presented at the Second Annual Convention of O.R. Society of India, held at Bombay, December, 1969.
- (vi) "Some remarks on the solution for the transportation problem", abstract in *Biometrics*, Vol. 27(2), June, 1971.

These papers have been prepared with author's name as G.V. Rao. In

addition to the above papers, the thesis contains a study of dynamic programming approach to a stochastic investment problem, a multi-stage inventory problem, the assignment problem, the critical path analysis and farm planning. The thesis also contains a general introduction giving the background of programming problems as well as brief summary of the work done by the previous authors and by the author on the subject. Appendices contain proofs of some results obtained in the thesis and some computer programs. These programs are written in FORTRAN language and are tested and run on IBM 1620 computer system attached to the University of Bombay. These are included so that the numerical examples considered in the thesis can be solved more efficiently with the help of a computer. Bibliography includes references to published work used in the thesis.

The author feels that the results obtained in this thesis contribute towards the advancement of our existing knowledge of Dynamic Programming Techniques in Operations Research. The results given in this thesis are not submitted to any other University for any degree or diploma.

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C O N T E N T S

	GENERAL INTRODUCTION	...	8
CHAPTER 1.	DYNAMIC PROGRAMMING TECHNIQUE IN PRODUCTION SCHEDULING AND EMPLOYMENT SMOOTHING PROBLEMS	...	44
	1.1 Introduction	...	44
	1.2 A deterministic demand model	...	45
	1.3 Formulation of the problem	...	46
	1.4 Dynamic programming approach	...	48
	1.5 Example	...	49
	1.6 Probabilistic demand model	...	54
	1.7 Solution of the problem	...	55
	1.8 Production scheduling problem with different types of production	...	57
	1.9 Dynamic programming approach	...	59
CHAPTER 2.	DYNAMIC PROGRAMMING AND RISK FUNCTIONS	...	63
	2.1 Introduction	...	63
	2.2 Sequential decision approach	...	65
	2.3 Dynamic programming approach	...	66
	2.4 Use of sufficient statistics	...	68
	2.5 Quadratic loss function	...	70
	2.6 Particular cases	...	71

CHAPTER 3. DYNAMIC PROGRAMMING TECHNIQUE AND OPTIMIZATION PROBLEMS ... 85

SECTION I : AN OPTIMAL DECISION PROCEDURE USING DYNAMIC PROGRAMMING TECHNIQUE ... 85

3.1.1 Introduction ... 85

3.1.2 Formulation of the problem ... 86

3.1.3 Dynamic programming approach ... 87

3.1.4 Solution of g when α and β are independently distributed ... 89

3.1.5 Particular cases ... 90

3.1.6 Numerical solution ... 93

3.1.7 Conclusion ... 94

SECTION II : OPTIMAL DECISION RULES FOR A STOCHASTIC INVESTMENT PROBLEM ... 97

3.2.1 Introduction ... 97

3.2.2 Structure of the problem ... 98

3.2.3 Dynamic programming approach ... 99

3.2.4 Use of discount factor ... 103

CHAPTER 4. DYNAMIC PROGRAMMING APPROACH TO INVENTORY TYPE PROBLEMS ... 107

SECTION I : THE WAREHOUSING PROBLEM ... 107

4.1.1 Introduction ... 107

4.1.2 Formulation of the model ... 108

4.1.3 Dynamic programming approach ... 109

4.1.4 Numerical example ... 110

4.1.5 Multiproduct model ... 112

4.1.6 Dynamic programming approach ... 114

4.1.7 Numerical example ... 115

	SECTION II : MULTISTAGE INVENTORY PROBLEM	...	118
	4.2.1 Introduction	...	118
	4.2.2 Formulation of the problem	...	118
	4.2.3 Dynamic programming approach	...	120
	4.2.4 Solution of the problem	...	122
	4.2.5 Example	...	126
CHAPTER 5.	DYNAMIC PROGRAMMING MODELS FOR JOB ASSIGNMENT, CRITICAL PATH LENGTH AND FARM PLANNING	...	128
	SECTION I : THE JOB ASSIGNMENT PROBLEM	...	128
	5.1.1 Introduction	...	128
	5.1.2 Mathematical formulation	...	129
	5.1.3 Dynamic programming approach	...	130
	5.1.4 Numerical example	...	131
	5.1.5 Some further results	...	133
	SECTION II : CRITICAL PATH LENGTHS IN NETWORK ANALYSIS	...	135
	5.2.1 Introduction	...	135
	5.2.2 Preliminaries	...	136
	5.2.3 Direct method for the determination of critical path	...	137
	5.2.4 Dynamic programming approach	...	138
	5.2.5 Numerical example	...	140

SECTION III : DYNAMIC PROGRAMMING AND FARM PLANNING	...	143
5.3.1 Introduction	...	143
5.3.2 Statement of the problem	...	144
5.3.3 Dynamic programming approach	...	145
5.3.4 Solution of the problem	...	147
5.3.5 Conclusion	...	149
APPENDICES.	...	150
APPENDIX 1. PROOFS OF SOME RESULTS	...	151
APPENDIX 2. SOME COMPUTER PROGRAMS	...	162
BIBLIOGRAPHY	...	174
SYNOPSIS	...	185