# CHAPTER 3

## DYNAMIC PROGRAMMING MODELS FOR JOB ASSIGNMENT, CRITICAL PATH LENTH AND FARM PLANNING

### SECTION 1 : THE JOB ASSIGNMENT PROBLEM

#### 3.1.1 INTRODUCTION

The assignment problem is just a special case of the transportation problem, but because of its simple strusture it can be solved more efficiently by its own solution procedure. The problem is to determine how the assignments should be made in order to minimize the total cost, given that each of the $n$ jobs (tasks) can be performed by any one of $n$ men (persons) and the cost of job $j$ being accomplished by person $i$ is $c_{ij}$. It is necessary that each job must be performed and that each person must be assigned to one and only one job. It can be seen that the mathematical formulation of the transportation problem reduces to the assignment problem when $m = n$, $a_i = 1$ for all $i$ and $b_j = 1$ for all $j$.

Variations of this problem, both mathematical and non-mathematical, have been known since a long time. However, Flood [41], Votaw and Orden [89] and others developed this problem with the help of linear programming technique during early 1950 s. Dantzig [26] and von Neumann [75] have discussed the computational advantages gained by considering this problem in combination with the dual linear programming technique. Kuhn [58] has developed a computationally efficient method using this duality and this method is known as Hungarian method for the assignment problem. The present study treats a general class of assignment problem using dynamic programming technique. Some modifications to the particular case when the number of jobs are not equal to the number of persons have also been given. However, simultaneously, in an article, Saksena [83] has given dynamic programming approach to this problem when (i) the number of jobs are equal to the number of persons, and (ii) the number of persons (m) are less than the number of jobs available (n) and the ratio n/m is an integer. We shall not make this assumption in our development of the procedure.

5.1.2 MATHEMATICAL FORMULATION

To formulate the assignment problem in the mathematical programming terms, let us define the activity variable, $x_{ij}$, as

$$x_{ij} = \begin{cases} 1 & \text{if job } j \text{ is performed by the } i^{th} \text{ person.} \\ 0 & \text{otherwise} \end{cases} \quad (5.1)$$

for $i = 1, \ldots, n$, and $j = 1, \ldots, n$ and let $c_{ij}$ be the cost of accomplishing the $j^{th}$ job by $i^{th}$ person. Then the optimization model is

$$\text{minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \; x_{ij} \quad (5.2)$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \quad , i = 1, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad , j = 1, \ldots, n \quad (5.3)$$

$$x_{ij} = 0 \text{ or } 1 \quad \text{for all } i \text{ and } j$$

If we apply the usual transportation technique to obtain the initial solution, then we can find that the solution so obtained is degenerate. This will always be the case in the assignment problem regardless of the method used to obtain the initial basis and moreover, the solution will continue to be degenerate at every iteration. Thus, the degeneracy technique should be used with the transportation technique until the optimal solution is obtained.

However, the special structure of the assignment problem helps to use a more convinient method of solution. One such method is based on the idea that the optimal solution remains the same if a constant is added (or subtracted) to any row or column of the cost matrix. This method of solution is known as the Hugarian method developed by Kuhn [58] based on a proof, by a Hungarian mathematician, Egervary [38], for a linear graph theorem of Konig [37].

### 3.1.3 DYNAMIC PROGRAMMING APPROACH

The assignment problem can also be solved by the functional equation approach of dynamic programming [11]. We shall, initially, consider the $n$ job, $n$ person problem and then generalise the approach to include all possible cases.

Consider the assignment problem in which $n$ jobs are to be performed by any of the $n$ persons so that each job is performed and that each person must be assigned to one and only one job. Given the cost, $c_{ij}$, of accomplishing the $j^{th}$ job by the $i^{th}$ person, it is necessary to find an optimal order of assignment so as to complete all the jobs at a minimum cost.

Let us define

$f(n)$ = the minimum cost for a n-job, n-person assignment problem, the assignment being made using an optimal policy.

Then, using the principle of optimality, we have

$$f(n) = \min_{j} \left\{ c_{j,n} + f(n-1) \right\} \tag{5.4}$$

with

$$f(o) = 0 \tag{5.5}$$

Using (5.4) and (5.5) we can obtain the solution for the assignment problem.

## 5.1.4 NUMERICAL EXAMPLE

To illustrate the dynamic programming approach to the assignment model, consider the problem with four jobs and four persons. The cost associated with assigning the $i^{th}$ person to the $j^{th}$ job is given in Table 5.1. The problem is to find an optimal assignment policy.

Table 5.1. Cost matrix.

| Job → / Person ↓ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 8 | 4 | 1 |
| 2 | 5 | 7 | 6 | 5 |
| 3 | 3 | 5 | 4 | 2 |
| 4 | 3 | 1 | 6 | 3 |

Solution :

$$f(1) = a_{11} = 1$$

$$f(2) = \begin{cases} f(1,2) = \min [1 + 7,\ 5 + 8] = 8 \\ f(1,3) = \min [1 + 5,\ 3 + 8] = 6 \\ f(1,4) = \min [1 + 1,\ 3 + 8] = 2 \\ f(2,3) = \min [5 + 5,\ 3 + 7] = 10 \\ f(2,4) = \min [5 + 1,\ 3 + 7] = 6 \\ f(3,4) = \min [3 + 1,\ 3 + 5] = 4 \end{cases}$$

$$f(3) = \begin{cases} f(1,2,3) = \min [c_{33} + f(1,2),\ c_{23} + f(1,3),\ c_{13} + f(2,3)] \\ \qquad = \min [4+8,\ 6+6,\ 4+10] = 12 \\ f(1,2,4) = \min [c_{43} + f(1,2),\ c_{23} + f(1,4),\ c_{13} + f(2,4)] \\ \qquad = \min [6+6,\ 6+2,\ 4+6] = 8 \\ f(1,3,4) = \min [c_{43} + f(1,3),\ c_{33} + f(1,4),\ c_{13} + f(3,4)] \\ \qquad = \min [6+6,\ 4+2,\ 4+4] = 6 \\ f(2,3,4) = \min [c_{43} + f(2,3),\ c_{33} + f(2,4),\ c_{23} + f(3,4)] \\ \qquad = \min [6+10,\ 4+6,\ 6+4] = 10 \end{cases}$$

$$f(4) = \{ f(1,2,3,4) = \min [c_{44} + f(1,2,3),\ c_{34} + f(1,2,4),\ c_{24} + f(1,3,4),\ c_{14} + f(2,3,4)]$$

$$= \min [3+12, 2+8,\ 5+6,\ 1+10] \} = 10$$

Hence the solution is

assign person 1 to job 1

person 2 to job 3

person 3 to job 4

person 4 to job 2

and the total minimum cost is 10 units.

5.1.5 SOME FURTHER RESULTS

Sometimes, there are situations where either the available men (m) are greater than the available jobs (n) or the available men are less than the available jobs. In such a situation some modifications to the earlier approach is needed.

Let us consider the case when the number of persons (m) available are more than the number of jobs (n), i.e., $m > n$, however, $m/n$ need not be an integer. The following assumptions are made to analyse the problem :

(i) no job is to be alloted to more than one person, and

(ii) all jobs are to be assigned to different persons.

Let $c_{ij}$ be the cost of job j being performed by person i, $i = 1, \ldots, m$ ; $j = 1, \ldots, n$. In this case, we have a rectangular matrix of order $m \times n$ representing the cost of accomplishing the n jobs by m persons. The first step now is to convert this matrix into a square matrix by introducing (m-n) dummy jobs. Since there is no cost associated for the accomplishment of these dummy jobs, the corresponding $c_{ij}$'s are all zero.

This reduces the problem to assign m men to m jobs so that the total cost of accomplishment of the jobs is minimum. We can now apply the procedure developed in section 5.1.3. The solution obtained in this case will indicate that (m-n) persons are not assigned to any jobs.

Similarly, if the number of persons(m) are less than the number of available jobs (n), then we introduce (n-m) dummy persons with associated zero costs. The solution obtained in this case will indicate that (n-m) jobs are not accomplished. However, if $n/m = \alpha$ is an integer (if not, introduce (n-m) dummy jobs so that $n'/m = \alpha'$ is an integer, where $n' = n + (n-m)$), as in [83], we can obtain a solution so that all the jobs are accomplished. (However, in this case some of the persons are assigned to $\alpha'$ jobs, i.e., more than one job.)

There are situations where it is necessary to assign given men to available jobs so as to maximise the total effectiveness or the total return. It is easily seen that, with some modifications, the technique developed in section 5.1.3. can be used to solve this problem.

## SECTION 2 : CRITICAL PATH LENTHS IN NETWORK ANALYSIS

### 2.1 INTRODUCTION

The technique of PERT is based on a simple concept called network logic, and using this logic any project can be represented in a pictorial form called network. The network becomes the basis for project planning, scheduling and follow-up [54,55,74]. In the PERT model such a network is viewed as representing a partial ordering of the many individual jobs that together comprise some project, the partial ordering is due to the requirement that all inward - pointing jobs at a node must be finished before any outward-pointing job at the node can be completed.

Drawing of network and analysing it consists of

(i) development of the project structure, i.e., visualising and listing the different activities to be performed and the sequence in which they are to be performed,

(ii) construction of the activity network representing the sequence and relationships of activities,

(iii) determination and assignment of time estimates for activities,

(iv) determination of the critical path, and

(v) matching the plan with objectives and drawing schedules for execution of activities.

Several authors have studied this problem of critical path analysis. Charnes, Charnes, et.al. [23] have given an approach using chance constrained and stochastic programming technique. Vajda [87] has given linear programming

...roach to this problem. Van Slyke's [88] approach is based on Monte ...o method. We shall use the functional equation approach of dynamic ...ramming to determine the critical path length in a network. A computer ...gram, in FORTRAN, is developed to solve the numerical example considered ...

... 2 PRELIMINARIES

The following terms which will be used very frequently in PERT ...alysis are defined below :

...NT. This is an inexplicitly identifiable point in time at which something ... happened or a situation has come into existance and the start coming into ...istance. There may be some amount of work or operation involved in reach... an event but an event by itself takes no time . An event shows only ... starting or end point of any activity. These are also called nodes, ...l stones, etc. Events are usually represented by circles.

...IVITY. This is clearly a definable task to which a known quantity of ...power and other resources have to be applied. An activity is represented ... n arrow, the head of which is directed towards the successor event. ... length of the arrow do not specify the magnitude of the activity.

...MY. A dummy is a dotted arrow which represents interrelationships but ...ries no time. Dummies are used for the completion of a logic in a network and is also used for bringing unique definition of activities with ...decessor and successor events.

NETWORK. A network is a diagram of a project plan. It consists of events and activities. A network usually starts with the initiation of the project and after many consecutive and simultaneous activities ends with the completion.

EXPECTED TIME. Basically, the system involves estimating the 'optimistic time', the 'most likely time', and the 'pessimistic time', and by means of a formula which is more emperical than scientific, produces an expected time.

EARLIEST EXPECTED TIME. The earliest expected time of an event is the earliest time or date by which that event can take place. This is obviously equal to the sum of all the activity expected times along the longest path leading from the starting event up to the event in question. This time shows when that particular event can be started after all the constraining activities leading to that event have been completed.

CRITICAL PATH. Critical path is the longest time path through a network and this represents the least time of achieving the objective without crashing. Thus, the critical path identifies activities which are crucial from the point of view of the completion of the project. Any delay or speeding up of those activities will affect the completion date of the project. Hence the critical path is that sequence of activities which will determine the total project time.

## ?.2.3 DIRECT METHOD FOR THE DETERMINATION OF CRITICAL PATH

It is clear from the definition of critical path that it is a path through the network which represents the earliest time of completing the project. Thus, we go through the network from the start to the finish of the project by computing when earliest a particular event (i.e., a certain level of achievement of the project) can be reached from the start of the project. The earliest that a certain event can be reached from the starting of the project is designated as earliest expected time for that event ($T_E$). The logic of the computation of the $T_E$ values for events in a network is understood, then we find that the $T_E$ for any event is the longest time on an activity converging on that event.

However, when the technolgical relationships are many and complicated, the solution of the problem by direct method becomes tidious. In such a situation, the solution can be obtained by the functional equation approach of dynamic programming [14,94].

5.4 DYNAMIC PROGRAMMING APPROACH

The problem here is to determine the longest time path through the network using the dynamic programming technique. Let us define

$L(I)$ = the length of the time path through the network from event I to N, I = 1, ..., N-1, using an optimal policy.

Employing the principle of optimality, we see that $L(I)$ satisfies the recurrence relation given by

$$L(I) = \max_J [t(I,J) + L(J)] \quad (5.6)$$

$$L(N) = 0 \quad (5.7)$$

...e $t(I,J)$ is the time required for the completion of an activity which ...ounded by two events I and J. These events are referred to as predessor ... succe.sor events for the activity.

Solution of equation (5.6) can be carried out as follows :

Define

$$L_1(I) = t(I, N)$$
$$J_1(I) = N , \quad \text{for all } I \tag{5.8}$$

...s $J(I)$ is the optimal path, and for $n \geq 2$ , the recurrence relation ...) can be written as

$$L_n(I) = \max_J \left[ t(I, J) + L_{n-1}(J) \right] \tag{5.9}$$

The value of $L_n(I)$ is calculated until $L_n(I)$ remains constant ...ntil $n = N-1$, since the convergence should take place with in $N-1$ steps. ...ltaneously we note down the optimal path, $J_n(I)$, for each I and for ...y value of n. In most of the cases, we can find that $L_n(I)$ remains ...tant for $n < N-1$. This value of $L_n(I)$ gives (for I = 1, since the ...ect is to be started from event 1) the earliest time of completing the ...ect and the corresponding path through the network is the critical path.

To solve a numerical problem, we shall prepare a table for the values ... $t(I, J)$. For a dummy activity bounded by events I and J, the value of ... J) is equal to zero. Since an event by itself takes no time $t(I,I)=0$. ...he technological relationship between events I and J do not exist, we ... dashes against the corresponding values of $t(I,J)$ indicating that it is not possible to move from event I to event J. This table will be in the form of a matrix of order N x N. This matrix is used to obtain the values of $f_2(I)$, $f_3(I)$, etc. with the help of equation (5.9).

### 5.2.3 NUMERICAL EXAMPLE

To illustrate the iterative procedure for finding the critical path length, we use the following example. However, a computer program, written in FORTRAN language, is given in Appendix so that the numerical examples can efficiently be solved with the help of a computer. The program has been tested and run on IBM 1620 computer system attached to the University.

Consider the network given in Figure 5.1. The activity durations (in weeks), t(I, J), are given in Table 5.2.

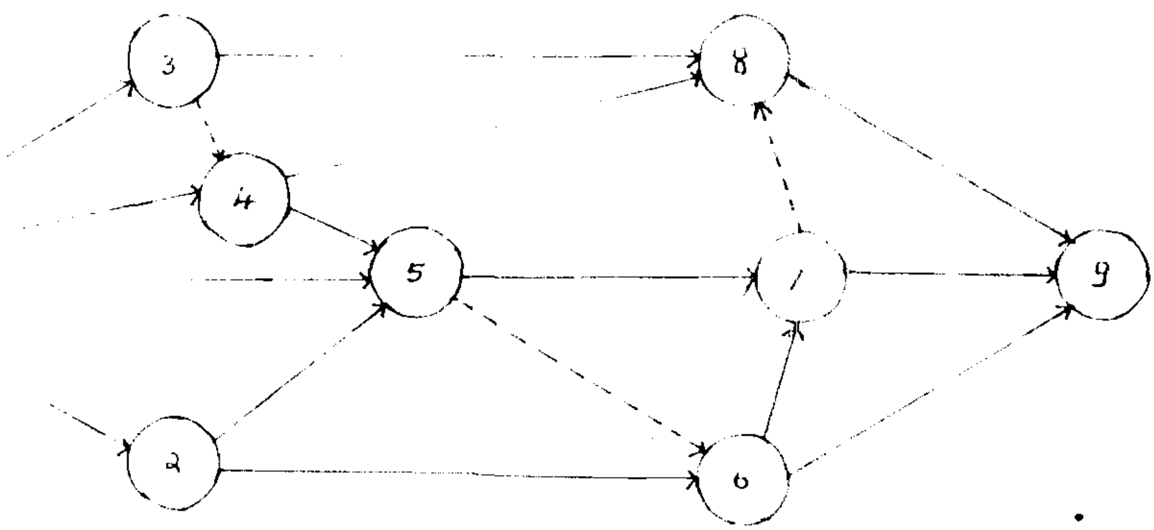


Fig. 5.1

Table 5.2. Activity durations.

| I \ J | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 5 | 4 | 3 | - | - | - | - |
| 2 | - | 0 | - | - | 7 | 6 | - | - | - |
| 3 | - | - | 0 | 0 | - | - | - | 9 | - |
| 4 | - | - | - | 0 | 2 | - | - | 5 | - |
| 5 | - | - | - | - | 0 | 0 | 7 | - | - |
| 6 | - | - | - | - | - | 0 | 4 | - | 4 |
| 7 | - | - | - | - | - | - | 0 | 0 | 6 |
| 8 | - | - | - | - | - | - | - | 0 | 13 |
| 9 | - | - | - | - | - | - | - | - | 0 |

Using equations (5.8) and (5.9), we obtain the following result (Table 5.3)

Table 5.3.

| I | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $f_1(I)$ | - | - | - | - | - | 4 | 6 | 13 | 0 |
| $d_1(I)$ | - | - | - | - | - | 9 | 9 | 9 | 9 |
| $f_2(I)$ | - | 10 | 22 | 18 | 13 | 10 | 13 | 13 | 0 |
| $d_2(I)$ | - | 5 | 8 | 8 | 7 | 7 | 8 | 9 | 9 |
| $f_3(I)$ | 27 | 20 | 28 | 18 | 20 | 17 | 13 | 13 | 0 |
| $d_3(I)$ | 3 | 5 | 8 | 8 | 7 | 7 | 8 | 9 | 9 |
| $f_4(I)$ | 27 | 27 | 22 | 22 | 20 | 17 | 13 | 13 | 0 |
| $d_4(I)$ | 3 | 5 | 8 | 5 | 7 | 7 | 8 | 9 | 9 |
| $f_5(I)$ | 29 | 27 | 22 | 22 | 20 | 17 | 13 | 13 | 0 |
| $d_5(I)$ | 2 | 5 | 4/8* | 5 | 7 | 7 | 8 | 9 | 9 |
| $f_6(I)$ | 29 | 27 | 22 | 22 | 20 | 17 | 13 | 13 | 0 |
| $d_6(I)$ | 2 | 5 | 4/8* | 5 | 7 | 7 | 8 | 9 | 9 |

* indicates 'either - or'

Since $f_6(I) = f_5(I)$, for all I, we discontinue our calculation.

Thus, the critical path is

$$1 \to 2 \to 5 \to 7 \to 8 \to 9$$

and the earliest expected time is 29 weeks.

## …TION 3 : DYNAMIC PROGRAMMING AND FARM PLANNING

### 3.1 INTRODUCTION

Farm planning deals with the application of economic principles to … organization and operations of farms. In farm planning, the objective … he cultivator is to maximise his net returns or farm business income … limited resources of land and capital (with a possible limitation on … our also). Thus, it is necessary to determine a best possible combination … choice of alternative crops.

Budgeting and programming [34,43,52] are the methods used to achieve … objective. Budgeting is a specific plan for the operation of the farm … ing some period of time. It takes into account a variable production … ram based upon the knowledge of the farm to be planned, its production … ibilities, the feasibility of carrying out alternate plans, etc. and … ests improved policies to the cultivator. This is done by comparing … expected income of these alternatives and selecting a policy which results … maximum return. There are several procedures involved in budgeting. … are : (i) recording an inventory of resources, (ii) forming price … ectations, (iii) preparing a crop plan taking into consideration the … lementary or the competetive aspect, (iv) estimating the yields, and … making a livestock plan taking into account the returns and the availa… …ty of land, labour and capital. Budgeting may be partial (for any … icular crop) or complete (taking the entire farm).

The recent development used as a device to specify an optimum organisation of resources and enterprices on farms and to suggest desirable farm adjustments is the programming method [34,43,52]. These approaches are based on linear programming technique. We shall consider here the dynamic programming approach to this problem with the restrictions on the availability of land and capital. However, the model can be extended to include some more restrictions to reflect actual farm conditions and to obtain more realistic solution.

## STATEMENT OF THE PROBLEM

Let us consider the following problem : A cultivator in a village has 22 acres of unirrigated land and the amount of cash available for cultivation is Rs. 54,000/- . The input-output data for the crop activities and the resource requirements are given in table 5.4. It is assumed that the soil can be made appropriate for these different crops by suitably adding fertilizers, chemicals, etc., the relevant cost being included in the capital requirement for cultivation. An optimum plan for cultivation is to be determined subject to the conditions of cash availability and the land available for cultivation.

Table 5.4. Input-Output data[1]

| Crops | Notation | Capital required per acre (in 000s of Rs.) | Value of output per acre (in 000s of Rs.) |
|---|---|---|---|
| Bajri | $P_1$ | 1.12 | 2.55 |
| Groundnut | $P_2$ | 3.76 | 5.80 |
| Bulga | $P_3$ | 1.17 | 1.60 |
| Jowar | $P_4$ | 1.01 | 3.42 |
| Wheat | $P_5$ | 3.53 | 3.42 |
| Gram | $P_6$ | 2.94 | 4.19 |
| Bajri-Gram | $P_7$ | 3.98 | 6.74 |
| Mug-Jowar | $P_8$ | 3.05 | 5.84 |

[1] taken from [34] with some modifications.

## 5.3.3 DYNAMIC PROGRAMMING APPROACH

To determine an optimum plan for cultivation, we proceed in the following way :

Let $A$ be the cash available for utilization and let $a_i$ be the cash requirement per acre for the $i^{th}$ crop, $i = 1, \ldots, N$. Let $L$ be the total acres of land available for cultivation. Let $g_i(x_i)$ be the profit (net return) from the $i^{th}$ crop when the $i^{th}$ crop is cultivated over an area of $x_i$ acres, $x_i$ being assumed to be an integer, $i = 1, \ldots, N$. Then the analytic problem reduces to that of maximizing the function

$$R(x_1, x_2, \ldots, x_N) = g_1(x_1) + g_2(x_2) + \ldots + g_N(x_N) \qquad (5.10)$$

subject to

$$\sum_{i=1}^{N} x_i \leq L$$

$$\sum_{i=1}^{N} R_i x_i \leq R \qquad (5.11)$$

$$x_i = 0, 1, \ldots, L$$

We now define $f_N(L,R)$ as the maximum income associated with an optimal choice of crops, when the total amount of expenditure do not exceed R units and the total land utilised for cultivation do not exceed L acres. Then we have

$$f_N(L,R) = \max R_N(x_1, x_2, \ldots, x_N) \qquad (5.12)$$

where the maximization is over the region of $x_i$ values defined by (5.11). Using the principle of optimality, the recurrence relation can be obtained as

$$f_N(L,R) = \max_{\{x_N\}} [\, g_N(x_N) + f_{N-1}(L-x_N, R-R_N x_N) \,] \qquad (5.13)$$

and

$$f_1(L,R) = g_1(x_1) \qquad (5.14)$$

where $x_1$ is taken over the region

$$0 \leq x_1 \leq \min \{ [\, L \,], [\, R/R_1 \,] \} \qquad (5.15)$$

where $[\, z \,]$ denotes the greatest integer less than or equal to $z$.

## .4 SOLUTION TO TH PROBLEM

Refering to the table 5.4, we can see that cultivation of Wheat ($P_5$) ults in a net loss of Rs.110/- per acre. So it is not advisable to ude Wheat for cultivation. Thus, we can see that there are seven sible budgets (given in table 5.5) with in the limitations of land and tal. From these alternative plans, we can select the plan - cultivate r ($P_4$) in an area of 22 acres and the corresponding farm income is 53,000/- approximately. However, this need not be the maximum possible ome within the given resources, since we can see here that the capital lly utilised. So we now apply the dynamic programming approach the combination of plans resulting in a maximum income.

Table 5.5. Feasible budgets.

| ops | Notation | Capital requirement/ acre (in ooos of Rs.) | Net income/ acre (in ooos of Rs.) | Total net income (in ooos of Rs.) | Land cultivated (acres) | Total investment (in ooos of Rs.) | Income-investment ratio |
|---|---|---|---|---|---|---|---|
| t | $P_1$ | 1.12 | 1.43 | 31.46 | 22 | 24.64 | 1.28 |
| oundnut | $P_2$ | 3.76 | 2.04 | 28.56 | 14 | 52.64 | 0.54 |
| | $P_3$ | 1.17 | 0.43 | 9.46 | 22 | 25.74 | 0.37 |
| ar | $P_4$ | 1.01 | 2.41 | 53.02 | 22 | 22.22 | 2.39 |
| m | $P_6$ | 2.94 | 1.25 | 22.50 | 18 | 52.92 | 0.43 |
| i-Gram | $P_7$ | 3.98 | 2.76 | 35.88 | 13 | 51.74 | 0.69 |
| -Jowar | $P_8$ | 3.05 | 2.79 | 47.43 | 17 | 51.85 | 0.92 |

Since the amount of cash available for cultivation is Rs. 54,000/- ... the land available is 22 acres, we consider combinations of crops which ...me maximum income - investment ratio and then apply dynamic programming ...ocedure. Since the combinations of $P_4$ and $P_1$ do not result in any ...ditional income, we consider combinations of $P_4$ and $P_8$. Using dynamic ...ogramming technique we find that (from table 5.6) the maximum income is ... 58,720/- and the optimum farm plan is to cultivate crop Jowar ($P_4$) in ...rea of 7 acres and crop Mug-Jowar ($P_8$) in an area of 15 acres. It is ...nd that any other combinations of crops will result in a lower income ... the income obtained from the above farm policy.

**Table 5.6. Optimal farm policy**

| ...nd culti-vated, $x_1$, ... $P_4$ ... | Net income $g_1(x_1)$ (in 000s of Rs.) | Amount of investment (in 000s of Rs.) | Land culti-vated, $x_2$, of $P_8$ (acres) | Total net income $f_2(L,R)$ (in 000s of Rs.) | Total amount of investment (in 000s of Rs.) |
|---|---|---|---|---|---|
| 1 | 2.41 | 1.01 | 17 | 49.84 | 52.86 |
| 2 | 4.82 | 2.02 | 17 | 52.25 | 53.87 |
| 3 | 7.23 | 3.03 | 16 | 51.87 | 51.83 |
| 4 | 9.64 | 4.04 | 16 | 54.28 | 52.84 |
| 5 | 12.05 | 5.05 | 16 | 56.69 | 53.85 |
| 6 | 14.46 | 6.06 | 15 | 56.31 | 51.81 |
| 7 | 16.87 | 7.07 | 15 | 58.72 | 52.82 |
| 8 | 19.28 | 8.08 | 14 | 58.34 | 50.79 |
| 9 | 21.69 | 9.09 | 13 | 57.97 | 48.74 |
| 10 | 24.10 | 10.10 | 12 | 57.98 | 46.70 |

## 3.5 CONCLUSION

The main difference between budgeting and programming is the mathematics behind it. Budgeting is usually used to determine one unique production program out of the many resulting in a maximum profit. Although other programming techniques, like linear programming and integer programming, can be used to obtain an optimum plan, we have demonstrated here the applicability of dynamic programming technique to this branch of planning.

We have, thus, enumerated the scope and application of dynamic programming technique to a diversified field of problems in Operations Research. However, the field of application of this technique is not exhaustive, and it is possible to find problems where dynamic programming technique can efficiently be applied.