APPENDICES

The appendices are divided into two parts : appendix 1 and appendix 2. Appendix 1 contains proofs of some results used in chapters 1 and 2. Appendix 2 contains some useful computer programs. These programs are written in FORTRAN language and are tested and run on IBM 1620 computer system attached to the Electronic Data Processing Centre of the University of Bombay. Results of some numerical examples considered in the thesis can more efficiently be obtained with the help of these programs and a computer.

APPENDIX 1. PROOFS OF SOME RESULTS

This appendix contains proofs of some results used in the thesis.

A.1.1 To derive equations (1.20) and (1.22).

From equation (1.19), we have

$$c_o + c(1 + \alpha) + h \int_o^x \phi(r)dr - p \int_x^\infty \phi(r)dr = o$$

$$\text{i.e.,} \quad c_o + c(1 + \alpha) - p + (h + p) \int_o^x \phi(r)dr = o$$

$$\text{i.e.,} \quad \int_o^x \phi(r)dr = \frac{p - c_o - c(1 + \alpha)}{h + p} \tag{A.1}$$

which is equivalent to equation (1.20).

We are given

$$\phi(r)dr = \lambda e^{-\lambda r} dr \tag{A.2}$$

Substituting (A.2) in (A.1), we get

$$\lambda \int_o^x e^{-\lambda r} dr = \frac{p - c_o - c(1 + \alpha)}{h + p}$$

$$\text{i.e.} \quad \lambda \left[\frac{e^{-\lambda r}}{-\lambda} \right]_o^x = \frac{p - c_o - c(1 + \alpha)}{h + p}$$

$$\text{i.e.,} \quad e^{-\lambda x} = 1 - \frac{p - c_o - c(1 + \alpha)}{h + p}$$

$$e^{\lambda x} = \frac{h + p}{h + c_o + c(1 + \gamma)} \tag{A.3}$$

which is equivalent to equation (1.22).

3. To obtain results of Section 2.6.1.

Let $x_1, x_2, \ldots, x_n$ be a random sample drawn from a normal population with mean α and unit variance.

$$\text{i.e.,} \quad dF(x \mid \alpha) = \frac{1}{\sqrt{2\pi}} e^{-(x-\alpha)^2/2} dx \tag{A.4}$$

where α is the unknown parameter whose prior distribution is also normal with mean μ (known) and unit variance.

$$\text{i.e.,} \quad dG_0(\alpha) = \frac{1}{\sqrt{2\pi}} e^{-(\alpha-\mu)^2/2} d\alpha \tag{A.5}$$

Let $s = \left(\sum_{i=1}^{n} x_i \right)/n$ be the sufficient statistics for the parameter α. Then s is $N(\alpha, {}^1/n)$. Hence the posterior distribution of α given s is

$$dG_n(\alpha \mid s,n) = \frac{dG_0(\alpha)\, d\,\Phi_n(s, \alpha)}{\int d\,\Phi_n(s,\alpha)\, dG_0(\alpha)}$$

$$= \frac{e^{-(\alpha-\mu)^2/2}\, e^{-\frac{n}{2}(s-\alpha)^2} d\alpha}{\int_\alpha e^{-\frac{n}{2}(s-\alpha)^2}\, e^{-(\alpha-\mu)^2/2} d\alpha}$$

$$= \frac{e^{-\frac{1}{2}\left[n(s-\alpha)^2 + (\alpha-\mu)^2\right]} d\alpha}{\int_\alpha e^{-\frac{n+1}{2}\left[\alpha - \frac{ns+\mu}{n+1}\right]^2 - \frac{n(s-\mu)^2}{2(n+1)}} d\alpha}$$

$$= \sqrt{\frac{n+1}{2\pi}}\, e^{-\frac{1}{2}(n+1)\left[\alpha - \frac{ns+\mu}{n+1}\right]^2} d\alpha \tag{A.6}$$

The posterior distribution of x_{n+1} given $x_1, x_2, \ldots, x_n$ is

$$dF_n(x_{n+1} \mid s,n) = \frac{\int_\alpha d\,\Phi_n(s,\alpha)\, dF(x_{n+1} \mid \alpha)\, dG_0(\alpha)}{\int_\alpha d\,\Phi_n(s,\alpha)\, dG_0(\alpha)}$$

$$\text{i.e., } dF_n(x_{n+1} \mid s,n) = \frac{\frac{1}{\sqrt{2\pi}} \int_\alpha e^{-\frac{n}{2}(s-\alpha)^2 - \frac{1}{2}(x_{n+1}-\alpha)^2 - \frac{1}{2}(\alpha-\mu)^2} d\alpha}{\int_\alpha e^{-\frac{n}{2}(s-\alpha)^2 - \frac{1}{2}(\alpha-\mu)^2} d\alpha}\, dx_{n+1}$$

$$\text{Numerator} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x_{n+1}^2}{2} - \frac{ns^2+\mu^2}{2} + \frac{(ns+x_{n+1}+\mu)^2}{2(n+2)}} \int e^{-\frac{n+2}{2}\left[\alpha - \frac{ns+x_{n+1}+\mu}{n+2}\right]^2} d\alpha$$

$$dF_n(x_{n+1} \mid s,n) = \frac{\frac{1}{\sqrt{2\pi}}\sqrt{\frac{2\pi}{n+2}}\; e^{-\frac{x_{n+1}^2}{2} - \frac{ns^2}{2} - \frac{\mu^2}{2} + \frac{(ns+x_{n+1}+\mu)^2}{2(n+2)}}}{\sqrt{\frac{2\pi}{n+1}}\; e^{-\frac{n(s-\mu)^2}{2(n+1)}}}\, dx_{n+1}$$

$$\text{i.e., } dF_n(x_{n+1} \mid s,n) = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{n+1}{n+2}}\; e^{-\frac{n+1}{2(n+2)}\left[x_{n+1} - \frac{ns+\mu}{n+1}\right]^2} dx_{n+1} \quad (A.7)$$

Hence, from (A.6), we see that the posterior distribution of the parameter α is normal with

$$\text{mean} = E(\alpha) = \frac{ns+\mu}{n+1} \quad (A.8)$$

and

$$\text{Variance} = V(\alpha) = \frac{1}{n+1} \quad (A.9)$$

Also, the posterior distribution of x_{n+1} is normal with mean $(ns + \mu) / (n+1)$ and variance $(n+2) / (n+1)$.

A.1.3 To obtain results of section 2.6.2.

Let $x_1, x_2, \ldots, x_n$ be a random sample drawn from the exponential distribution

$$dF(x|\theta) = \frac{1}{\theta} e^{-x/\theta} \, dx , \qquad x \geq 0, \quad \theta > 0 \qquad (A.10)$$

where θ is the unknown parameter whose prior distribution is

$$dG_o(\theta) = \frac{\mu^{\nu}}{\Gamma(\nu)} \quad \frac{e^{-\mu/\theta}}{\theta^{\nu+1}} \, d\theta \qquad (A.11)$$

where μ and ν are known constants. The prior density of the form (A.11) is known as inverted gamma density function.

Let $s = \sum_{i=1}^{n} x_i$ be the sufficient statistic for θ. Then, the posterior distribution of θ given s is

$$dG_n(\theta \mid s,n) = \frac{dG_o(\theta) \, d\,\Phi_n(s,\theta)}{\int d\,\Phi_n(s,\theta) \, dG_o(\theta)}$$

$$= \frac{\dfrac{e^{-\mu/\theta}}{\theta^{\nu+1}} \; \dfrac{s^{n-1}}{\Gamma(n)} \; \dfrac{e^{-s/\theta}}{\theta^{n}} \, d\theta}{\int_{\theta} \dfrac{s^{n-1}}{\Gamma(n)} \; \dfrac{e^{-s/\theta}}{\theta^{n}} \; \dfrac{e^{-\mu/\theta}}{\theta^{\nu+1}} \, d\theta}$$

$$= \frac{(s+\mu)^{n+\nu}}{\Gamma(n+\nu)} \quad \frac{e^{-(s+\mu)/\theta}}{\theta^{n+\nu+1}} \, d\theta \qquad (A.12)$$

This distribution also belongs to the inverted gamma density function.

The posterior distribution of x_{n+1} given $x_1, \ldots, x_n$ is

$$dF_n(x_{n+1} \mid s,n) = \frac{\int d\Phi_n(s,\theta)\, dF(x_{n+1} \mid \theta)\, dG_0(\theta)}{\int d\Phi_n(s,\theta)\, dG_0(\theta)}$$

$$= \frac{\int_\theta \frac{s^{n-1}}{\Gamma(n)} \frac{e^{-s/\theta}}{\theta^n} \frac{1}{\theta} e^{-x_{n+1}/\theta} \frac{e^{-\mu/\theta}}{\theta^{\nu+1}} d\theta}{\int_\theta \frac{s^{n-1}}{\Gamma(n)} \frac{e^{-s/\theta}}{\theta^n} \frac{e^{-\mu/\theta}}{\theta^{\nu+1}} d\theta} dx_{n+1}$$

$$= \frac{\Gamma(n+\nu+1)}{(x_{n+1}+s+\mu)^{n+\nu+1}} \frac{(s+\mu)^{n+\nu}}{\Gamma(n+\nu)} dx_{n+1}$$

$$= \frac{\Gamma(n+\nu+1)}{\Gamma(n+\nu)} \frac{\frac{1}{s+\mu}}{\left(1+\frac{x_{n+1}}{s+\mu}\right)^{n+\nu+1}} dx_{n+1} \qquad (A.13)$$

To find the mean and variance of the posterior distribution of θ.

$$E(\theta) = \int_\theta \theta \frac{(s+\mu)^{n+\nu}}{\Gamma(n+\nu)} \frac{e^{-(s+\mu)/\theta}}{\theta^{n+\nu+1}} d\theta$$

$$= \frac{(s+\mu)^{n+\nu}}{\Gamma(n+\nu)} \int_\theta \frac{e^{-(s+\mu)/\theta}}{\theta^{n+\nu}} d\theta$$

$$= \frac{s+\mu}{n+\nu-1} \qquad (A.14)$$

$$E(\theta^2) = \int_\theta \theta^2 \frac{(s+\mu)^{n+\nu}}{\Gamma(n+\nu)} \frac{e^{-(s+\mu)/\theta}}{\theta^{n+\nu+1}} \, d\theta$$

$$= \frac{(s+\mu)^{n+\nu}}{\Gamma(n+\nu)} \int_\theta \frac{e^{-(s+\mu)/\theta}}{\theta^{n+\nu-1}} \, d\theta$$

$$= \frac{(s+\mu)^2}{(n+\nu-1)\ (n+\nu-2)}$$

$$V(\theta) = E(\theta^2) - [E(\theta)]^2$$

$$= \frac{(s+\mu)^2}{(n+\nu-1)^2\ (n+\nu-2)} \qquad (A.15)$$

A.1.4 To obtain results of section 2.6.3.

Let $x_1, x_2, \ldots, x_n$ be a random sample drawn from the Poisson distribution

$$dP(x \mid \theta) = \frac{e^{-\lambda} \lambda^x}{x!} \qquad (A.16)$$

where λ is the unknown parameter whose prior distribution is

$$dG_o(\lambda) = \frac{\alpha(\alpha\lambda)^{\beta-1}}{\Gamma(\beta)} e^{-\alpha\lambda} d\lambda \qquad (A.17)$$

where α and β are known constants. Let $s = \sum_{i=1}^{n} x_i$ be the sufficient statistic for the parameter λ. Then the posterior distribution of λ

given s is

$$dG_n(\lambda \mid s,n) = \frac{dG_o(\lambda)\, d\Phi_n(s,\lambda)}{\int d\Phi_n(s,\lambda)\, dG_o(\lambda)}$$

$$= \frac{\lambda^{s+\beta-1} e^{-(n+\alpha)\lambda} d\lambda}{\int_\lambda \lambda^{s+\beta-1} e^{-(n+\alpha)\lambda} d\lambda}$$

$$= \frac{(n+\alpha)^{s+\beta}}{\Gamma(s+\beta)} e^{-(n+\alpha)\lambda} \lambda^{s+\beta-1} d\lambda \qquad (A.18)$$

The posterior distribution of x_{n+1} given $x_1, \ldots, x_n$ is

$$dF_n(x_{n+1} \mid s,n) = \frac{\int d\Phi_n(s,\lambda)\, dF(x_{n+1} \mid \lambda)\, dG_o(\lambda)}{\int d\Phi_n(s,\lambda)\, dG_o(\lambda)}$$

$$= \frac{\frac{1}{x_{n+1}!}\int_\lambda e^{-(n+\alpha+1)\lambda} \lambda^{s+x_{n+1}+\beta-1} d\lambda}{\int_\lambda e^{-(n+\alpha)\lambda} \lambda^{s+\beta-1} d\lambda}$$

$$= \frac{\Gamma(s+x_{n+1}+\beta)}{\Gamma(s+\beta)} \left(\frac{n+\alpha}{n+\alpha+1}\right)^{s+\beta} \frac{1}{x_{n+1}!} \frac{1}{(n+\alpha+1)^{x_{n+1}}} \qquad (A.19)$$

which belongs to the negative binomial density function.

The mean and the variance of the posterior distribution of λ are

$$E(\lambda) = \int_\lambda \lambda \frac{(n+\alpha)^{s+\beta}}{\Gamma(s+\beta)} e^{-(n+\alpha)\lambda} \lambda^{s+\beta-1} d\lambda$$

$$= \frac{(n+\alpha)^{s+\beta}}{\Gamma(s+\beta)} \int_\lambda e^{-(n+\alpha)\lambda} \lambda^{s+\beta} d\lambda$$

$$= \frac{(n+\alpha)^{s+\beta}}{\Gamma(s+\beta)} \frac{\Gamma(s+\beta+1)}{(n+\alpha)^{s+\beta+1}}$$

$$= \frac{s+\beta}{n+\alpha} \qquad (A.20)$$

$$E(\lambda^2) = \frac{(n+\alpha)^{s+\beta}}{\Gamma(s+\beta)} \int_\lambda e^{-(n+\alpha)\lambda} \lambda^{s+\beta+1} d\lambda$$

$$= \frac{(s+\beta)(s+\beta+1)}{(n+\alpha)^2}$$

$$V(\lambda) = E(\lambda^2) - [E(\lambda)]^2$$

$$= \frac{(s+\beta)(s+\beta+1)}{(n+\alpha)^2} - \frac{(s+\beta)^2}{(n+\alpha)^2}$$

$$= \frac{s+\beta}{(n+\alpha)^2} \qquad (A.21)$$

A.1.5 To obtain results of section 2.6.4.

Let $x_1, x_2, \ldots, x_n$ be a random sample drawn from the binomial distribution

$$dF(x \mid \lambda) = \binom{N}{x} \lambda^{x} (1-\lambda)^{N-x} \tag{A.22}$$

where is the unknown parameter whose prior distribution is of the form

$$dG_0(\lambda) = \frac{\lambda^{\alpha-1}(1-\lambda)^{\beta-1}}{B(\alpha,\beta)}\, d\lambda \tag{A.23}$$

where α and β are constants. Let $s = \sum_{i=1}^{n} x_i$ be the sufficient statistic for the parameter λ. Then the posterior distribution of given s is

$$dG_n(\lambda \mid s,n) = \frac{dG_0(\lambda)\, d\Phi_n(s,\lambda)}{\int dG_0(\lambda)\, d\Phi_n(s,\lambda)}$$

$$= \frac{\lambda^{s}(1-\lambda)^{Nn-s}\lambda^{\alpha-1}(1-\lambda)^{\beta-1}\, d\lambda}{\int \lambda^{s}(1-\lambda)^{Nn-s}\lambda^{\alpha-1}(1-\lambda)^{\beta-1}\, d\lambda}$$

i.e., $$dG_n(\lambda \mid s,n) = \frac{\lambda^{s+\alpha-1}(1-\lambda)^{Nn+\beta-s-1}\, d\lambda}{B(\alpha+s,\, Nn+\beta-s)} \tag{A.24}$$

The posterior distribution of x_{n+1} given $x_1, \ldots, x_n$ is

$$dF_n(x_{n+1} \mid s,n) = \frac{\int d\,\Phi_n(s,\lambda)\; dF(x_{n+1} \mid \lambda)\; dG_o(\lambda)}{\int d\,\Phi_n(s,\lambda)\; dG_o(\lambda)}$$

$$= \frac{\binom{N}{x_{n+1}} \int_\lambda \lambda^{s}(1-\lambda)^{Nn-s}\, \lambda^{x_{n+1}}(1-\lambda)^{N-x_{n+1}}\, \lambda^{\alpha-1}(1-\lambda)^{\beta-1}\, d\lambda}{\int_\lambda \lambda^{s}(1-\lambda)^{Nn-s}\, \lambda^{\alpha-1}(1-\lambda)^{\beta-1}\, d\lambda}$$

$$= \binom{N}{x_{n+1}} \frac{B(s+\alpha+x_{n+1},\; N(n+1)+\beta-s)}{B(s+\alpha,\; Nn+\beta-s)} \qquad \text{(A.25)}$$

The mean and the variance of the posterior distribution of λ are

$$E(\lambda) = \int_\lambda \lambda \frac{\lambda^{s+\alpha-1}(1-\lambda)^{Nn+\beta-s-1}}{B(\alpha+s,\; Nn+\beta-s)}\, d\lambda$$

$$= \frac{B(\alpha+s+1,\; Nn+\beta-s)}{B(\alpha+s,\; Nn+\beta-s)}$$

$$= \frac{\alpha+s}{Nn+\alpha+\beta} \qquad \text{(A.26)}$$

$$E(\lambda^2) = \int_{\lambda} \frac{\lambda^2 \lambda^{s+\alpha-1} (1-\lambda)^{Nn+\beta-s-1}}{B(\alpha+s,\ Nn+\beta-s)} d\lambda$$

$$= \frac{B(\alpha+s+2,\ Nn+\beta-s)}{B(\alpha+s,\ Nn+\beta-s)}$$

$$= \frac{(\alpha+s)(\alpha+s+1)}{(Nn+\beta+\alpha)(Nn+\beta+\alpha+1)}$$

$$V(\lambda) = E(\lambda^2) - [E(\lambda)]^2$$

$$= \frac{(\alpha+s)(\alpha+s+1)}{(Nn+\beta+\alpha)(Nn+\beta+\alpha+1)} - \frac{(\alpha+s)^2}{(Nn+\beta+\alpha)^2}$$

$$= \frac{(\alpha+s)(Nn+\beta-s)}{(Nn+\beta+\alpha)^2(Nn+\beta+\alpha+1)} \qquad (A.27)$$

APPENDIX 2. SOME COMPUTER PROGRAMS

This appendix contains some computer programs (written in FORTRAN language) which are used to solve the numerical examples considered in the thesis. These programs have been tested and run on IBM 1620 computer system attached to the University of Bombay.

A.2.1 PROGRAM FOR THE PROBLEM OF CHAPTER 1

This program is written for the deterministic production scheduling and employment smoothing problem. The program is as follows :

```
C        FORTRAN PROGRAM FOR THE DETERMINISTIC PRODUCTION-SCHEDULING
         AND EMPLOYMENT SMOOTHING PROBLEM USING D.P. TECHNIQUE.
         DIMENSION  R(10), FX(10), FKX(10), X(10), XO(10), FM(10), FL(10)
         READ  116, N, M, CO, C1, C2, H, P, (R(J), J = 1, N), XO1
         DO  4 I = 1,M
    4    FKX(I) = 0.0
         J = 1
         DO  10  I = 1,M
         X(I) = I
         IF(X(I) - R(J)) 15, 15, 20
   15    VT = P*(R(J) - X(I))
         GO  TO  25
   20    VT = H*(X(I) - R(J))
   25    IF(X(I) - XO1) 30, 30, 35
   30    GT = C2*(XO1 - X(I))
         GO  TO  10
```

```
35    GT = C1*(X(I) - X01)
10    FX(I) = FKX(I) + CO* X(I) + VT + GT
      PRINT  102, J, (X(I), I = 1, M), (FX(I), I = 1, M), X01
      PRINT  104
      DO  5  I = 1, M
 5    FKX(I) = FX(I)
      DO  17  J = 2, N
      PRINT  103, J
      PRINT  104
      PRINT  107
      PRINT  104
      DO  19  I = 1, M
      X(I) = I
      IF(X(I) - R(J)) 115, 115, 120
115   VT = P*(R(J) - X(I))
      GO  TO  125
120   VT = H*(X(I) - R(J))
125   DO  140  K = 1, M
      XO(K) = K
      IF(X(I) - XO(K)) 130, 130, 135
130   GT = C2*(XO(K) - X(I))
      GO  TO  160
135   GT = C1*(X(I) - XO(K))
160   FM(K) = FKX(K) + CO* X(I) + VT + GT
140   CONTINUE
```

```
      TEMPX = FM(1)
      M1 = 1
      DO 220 K = 2,M
      IF(TEMPX - FM(K)) 220, 220, 210
210   TEMPX = FM(K)
      M1 = K
220   CONTINUE
      FX(I) = TEMPX
      XO(I) = M1
      PRINT 106, X(I), (FM(K), K = 1, M), FX(I), XO(I)
 19   CONTINUE
      PRINT 104
      DO 155 I = 1, M
155   FKX(I) = FX(I)
 17   CONTINUE
102   FORMAT(1H1, 5X, 6HPERIO D, 5X, I2 // 5X, 4HXNEW, 5X, 9(4X, F6.2) //
     1  5X, 5HFX(I), 4X, 9(2X, F8.2) // 5X, 4HXOLD, 9 X, F6.2)
103   FORMAT( / 5X, 6HPERIOD, I4 /)
104   FORMAT(5X, 99(1H-))
106   FORMAT(5X, I2, 3X, 9F8.2, 2X, F8.2, 6X, I2)
107   FORMAT(5X, 4HXNEW, 20X, 5HFX(I), 58X, 4HXOLD)
116   FORMAT(2I4, 5F6.2/4F6.2, 5X,F4.2)
      END
```

This program is suitable for a problem with a maximum demand of 10 units and a maximum of 10 periods. However, by making suitable changes in the DIMENSION, READ, PRINT and FORMAT statements, the problems of higher order can be solved with the help of the same program.

.2.2 PROGRAM FOR THE PROBLEM OF CHAPTER 2

This program is written for obtaining a decision procedure for exponential distribution. The program is as follows :

```
      FORTRAN PROGRAM FOR THE RISK FUNCTION USING D.P. TECHNIQUE
50    READ 105, AK, UM, UN, C
      AN = 1
7     IF(C-(AK* UM** 2)/((UN-1.)** 2* (AN+UN-2.))) 10, 15, 15
10    AN = AN + 1.
      GO TO 7
15    PRINT 106, AK, UM, UN, C, AN
      PRINT 108
      PRINT 109
      PRINT 111
      PRINT 109
      A1 = 0.005
45    A1 = A1+C
      A2 = AK* UM** 2/((UN-1.)** 2*(AN+UN-2.))
      IF(A2) 20, 18, 18
18    IF(A1-A2) 20, 30, 25
```

```
25    PRINT  112, AN, A1, A2, A2
      GO  TO  35
20    PRINT  113, AN, A1, A2, A1
      GO  TO  35
30    PRINT  114, AN, A1, A2, A2
35    A1 = A2
      AN = AN-1
      IF(AN) 40, 45, 45
40    PRINT  109
      PRINT  116
      GO  TO  50
105   FORMAT(4F6.2)
106   FORMAT(1H1, 5X, 2HK =, F6.1, 5X, 3HMU =, F6.1, 5X, 3HNU =,F6.2, 5X,
    1 2HC =, F6.2, 5X, 2HN =, F6.2/)
108   FORMAT(/5X, 47HDECISION PROCEDURE FOR EXPONENTIAL DISTRIBUTION/)
109   FORMAT(5X, 75(1H-))
111   FORMAT(5X,1HN,5X, 10HI=C+F(N+1), 5X, 30HII=K*MU** 2/((NU-1)**2
    1 *(N+NU-2)), 5X, 9HMIN(I,II), 5X, 8HDECISION)
112   FORMAT(5X, F4.1, 5X, F6.2, 18X, F6.2, 12X, 1HT)
113   FORMAT(5X, F4.1, 5X, F6.2, 19X, F6.2, 18X, F6.2, 12X, 1HC)
114   FORMAT(5X, F4.1, 5X, F6.2, 19X, F6.2, 18X, F6.2, 12X, 3HT/C)
116   FORMAT(5X, 42HT = TERMINATE AND TAKE G*(X) = (S+MU)/(N+NU-2)/
    1 5X, 32HC = CONTINUE FURTHER WITH A SAMPLE)
      END
```

The following notations are used here for the purpose of writing the program :

AK = K

MU = μ

NU = ν

AN = N

The GO TO 50 statement in the above program is included to help to obtain results for different sets of values of K, μ, ν and C. The same program can be used to obtain results for different distributions by making suitable changes in the statements such as READ, PRINT, FORMAT, statement numbered 7 and the statement for calculating the value of A2.

7.2.3 PROGRAM FOR THE PROBLEM OF CHAPTER 4

The program is written for the single - product warehousing problem. The program is as follows :

```
C         FORTRAN PROGRAM FOR SINGLE - PRODUCT WAREHOUSING PROBLEM
          DIMENSION  XX(12), YY(12), S(12), P(12), R(12), D(12)
          READ  101, (S(I), P(I), I = 1,12)
          READ  104, H, W
          PRINT  103
          FKX = 0.0
          DO  5  I = 1,12
          PRINT  106, I
          XM = 0.0
```

```
10    B = 0.0
      R(I) = 0.0
      D(I) = 0.0
      XX(I)= 0.0
      YY(I)= 0.0
15    FX = 0.0
      FX = FX + YY(I)*(S(I)+H) - XX(I)*(P(I) + H) + FKX
      PRINT  105, FX, YY(I), XX(I)
      IF(FX-B) 25, 25, 20
20    B = FX
      R(I) = XX(I)
      D(I) = YY(I)
25    XX(I) = XX(I) + 1.0
      YY(I) = YY(I) + 1.0
      IF(XX(I) - XM) 30, 30, 35
30    IF(YY(I) - XM) 15, 15, 35
35    FX = B
      XX(I) = R(I)
      YY(I) = D(I)
      XM = XM + 1.0
      IF(XM-W) 10, 10, 40
40    PRINT  102, FX, XX(I), YY(I)
      FKX = FX
 5    CONTINUE
101   FORMAT (2F6.2)
```

```
102    FORMAT(5X, 15HOPTIMAL PROFIT = , F10.4/5X, 26HOPTIMAL PURCHASE
      1 QUANTITY = , F6.2/5X, 25HOPTIMAL SELLING QUANTITY =, F6.2)
103    FORMAT (5X, 35HSOLUTION OF THE WAREHOUSING PROBLEM)
104    FORMAT(2F4.0)
105    FORMAT(5X, 3F8.2)
106    FORMAT (/5X, 6H PERIOD, 5X, I4 /)
       END
```

This program has been written for a 12 period warehousing problem. However, with suitable changes in statements such as DIMENSION, READ, and FORMAT and in Do - loop, the problems of higher order can be solved using the same program.

.2.4 PROGRAM FOR THE PROBLEM OF CHAPTER 5 SECTION 2

The program for determining the critical path length using dynamic programming technique is given below :

```
       DIMENSION  T(25,25), F1(25), FM(25), T1(25, 25), AI(25), AJ(25)
       READ  101, N, ((T(I,J), J = 1,N), I = 1,N)
       PRINT 113
       DO  9  I = 1, N
    9  PRINT  111, (T(I,J), J = 1,N)
       PRINT  106
       DO  5 I = 1,N
       J = N
       K = I
```

```
      IF(T(I,J) + 1.0) 10, 15, 10
15    F1(K) = - 1.0
      J1 = - 1
      GO  TO  20
10    F1(K) = T(I,J)
      J1 = J
20    AI(K) = I
      AJ(K) = J1
 5    CONTINUE
      M = 1
      PRINT  107, M
      PRINT  108
      PRINT  102, (AI(K), K = 1,N)
      PRINT  108
      PRINT  103, (F1(K), K = 1,N)
      PRINT  108
      PRINT  104, (AJ(K), K = 1,N)
100   DO 21 I = 1,N
      DO 21 J = 1,N
21    T1(I,J) = 0.0
      DO 25 I = 1,N
      K=1
      DO  30  J = 1,N
      IF (T(I,J) + 1.0) 35, 40, 35
35    IF(F1(K) + 1.0) 45, 40, 45
```

```
45    T1(I,J) = F1(K) + T(I,J)
      GO  TO  42
40    T1(I,J) = -1.0
42    K = K+1
      IF(K-N) 30, 30, 25
30    CONTINUE
25    CONTINUE
      DO  50  I = 1,N
      K = I
      J = 1
71    IF(T1(I,J) + 1.0) 61, 62, 61
62    J = J + 1
      IF(J-N) 71, 71, 72
72    FM(K) = -1.0
      J2 = -1.
      GO  TO  56
61    FM(K) = T1(I,J)
      J1 = J
      DO  55  J = J1,N
      IF(FM(K) - T1(I,J)) 60, 60, 55
60    FM(K) = T1(I,J)
      J2 = J
55    CONTINUE
56    AJ(K) = J2
      AI(K) = I
```

```
50    CONTINUE
      M = M + 1
      PRINT 107, M
      PRINT  108
      PRINT  102,(AI(K), K = 1,N)
      PRINT  108
      PRINT  103,(FM(K), K = 1,N)
      PRINT  108
      PRINT  104,(AJ(K), K=1,N)
      PRINT  108
      K = 1
85    IF(FM(K) - F1(K)) 80, 75, 80
75    K = K+1
      IF(K-N) 85, 85, 90
80    K = 1
81    F1(K) = FM(K)
      K = K+1
      IF(K-N) 81, 81, 95
95    N2 = N-2
      IF(M-N2) 100, 100, 90
90    STOP
101   FORMAT (I4/(20F4.1)
102   FORMAT (5X, 1HI, 5X, 14F6.0/)
103   FORMAT (6X, 5HFM(K), 14F6.1/)
104   FORMAT (5X, 1HJ, 5X, 14F6.0)
```

```
106   FORMAT (5X, 28HSOLUTION OF THE PERT PROBLEM)
107   FORMAT (/// 5X, 16 HITERATION NUMBUR, 5X, I4)
108   FORMAT (5X, 90(1H-))
111   FORMAT (5X, 14F6.1)
113   FORMAT (5X, 12HGIVEN MATRIX)
      END
```

A network with a maximum of 25 events can be solved with this program. Suitable modifications in DIMENSION, READ, PRINT and FORMAT statements will help to solve the problems of higher dimensions. Further, the capacity restriction of the computer is to be taken into account while introducing these modifications.