A STUDY OF SOME IDENTITIES STATED BY SRINIVASA RAMANUJAN IN HIS 'LOST' NOTEBOOK AND EARLIER WORKS

A THESIS SUBMITTED TO THE UNIVERSITY OF MYSORE
FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

BY

CHANDRASHEKAR ADIGA

DEPARTMENT OF POST- GRADUATE STUDIES'
AND RESEARCH IN MATHEMATICS
UNIVERSITY OF MYSORE
MYSORE-570006

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DECLARATION

I hereby declare that the thesis entitled "A STUDY OF SOME IDENTITIES STATED BY SRINIVASA RAMANUJAN IN HIS 'LOST' NOTEBOOK AND EARLIER WORKS" submitted to the University of Mysore for the degree of "DOCTOR OF PHILOSOPHY" is the result of the work carried out by me in the Department of Post-graduate Studies and Research in Mathematics, Manasagangotri, Mysore, under the guidance of Professor S. Bhargava during the period 1980-1983.

I further declare that the results of this work have not been submitted previously to this or any other university for any degree, diploma or fellowship.

Adiga.

(CHANDRASHEKAR ADIGA)

CERTIFICATE

Tnis is to certify that the thesis entitled, "A **STUDY OF** SOME IDENTITIES STATED **BY** SRINIVASA RAMANUJAN IN HIS 'LOST' NOTEBOOK AND **EARLIER** WORKS" submitted by Shri Chandrashekar Adiga for the degree of "DOCTOR OF PHILOSOPHY" of the University **of** Mysore is the result of research work done by him under my guidance during the period 1980-1983. I also certify that this work has not been submitted by him to this or to any other university for any degree, diploma or fellowship.

S. Bhargava

(S. BHARGAVA)
Guide.

Dr. S. Bhargava
Professor and Head of
Department of Postgraduate Studies and
Research in Mathematics
University of Mysore,
MYSORE - 570 006.

PREFACE

This thesis consists of six chapters being my research work carried out in the Department of Post-graduate Studies and Research in Mathematics, Manasagangotri, University of Mysore, Mysore, since the year 1980, It is a study of parts of Srinivasa Ramanujan's work concerning basic hypergeometric transformations including q-continued fraction developments and the theory of theta-functions. Chapter I is a brief survey of relevant literature with a summary of my results presented in the thesis. Chapter II is a study of some q-continued fraction expansions listed in Ramanujan's 'lost notebook'. Chapter III deals with generalizations of some results of Chapter II, and Chapter IV, with a 'remarkable formula' of Ramanujan of which Jacobi's triple product identity and q-binomial theorem are special cases. It also has numerous other applications in the theory of theta-functions. In Chapters V and VI we make a study of parts of the theory of theta-functions developed by Ramanujan in his 'second notebook', Chapter V dealing with some interesting identities deducible directly from Ramanujan's definitions and Chapter VI with a number of interesting applications of above mentioned 'remarkable formula' of Ramanujan.

In undertaking this work related to the famous Indian mathematician, I was stimulated by some of the recent papers of Bruce Berndt [Mathematics Magazine, Vol.51, 1978],

George Andrews [American Mathematical Monthly, Vol.86, 1979], Richard Askey [American Mathematical Monthly, Vol.87, 1980] and K. Venkatachaliengar [Presidential address, Forty-sixth Conference of the Indian Mathematical Society, 1980] in which they have expounded -the profundity and relevance of Ramanujan's works and the need for continued investigation.

This research work was done under the direction of Professor S. Bhargava and I express my deep sense of gratitude to him for his guidance all through the work. I am also highly tbankful to Professor Bruce Berndt of University of Illinois, U.S.A., for his many valuable suggestions.

I take this opportunity to thank the University Grants Commission for awarding me a research fellowship during the years 1980-83 and the University of Mysore for providing me necessary research facilities.

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CHANDRASHEKAR ADIGA

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